# Unit 11: Surds and Trigonometry

@whisto_maths

# Trigonometry

## What do I need to be able to do?

By the end of this unit you should be able to:

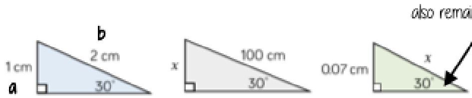
- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

## Keywords

**Enlarge:** to make a shape bigger (or smaller) by a given multiplier (scale factor)
**Scale Factor:** the multiplier of enlargement
**Constant:** a value that remains the same
**Cosine ratio:** the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
**Sine ratio:** the ratio of the length of the opposite side to that of the hypotenuse.
**Tangent ratio:** the ratio of the length of the opposite side to that of the adjacent side.
**Inverse:** function that has the opposite effect.
**Hypotenuse:** longest side of a right-angled triangle. It is the side opposite the right-angle.

## Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same




a : b
1 : 2

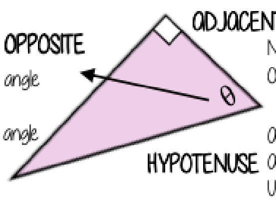
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

## Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way




OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

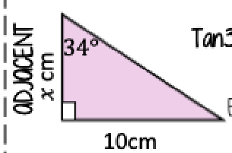
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

## Tangent ratio: side lengths

$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



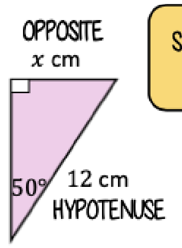

$\tan 34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \tan 34 = 10$

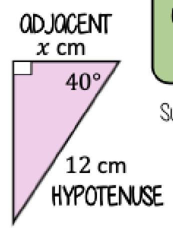
$x = \frac{10}{\tan 34} = 14.8\text{cm}$

## Sin and Cos ratio: side lengths




$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The $\sin(x)$ ratio is the same as the $\cos(90-x)$ ratio

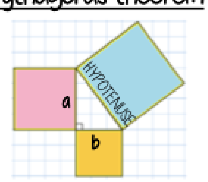



$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

## Pythagoras theorem R



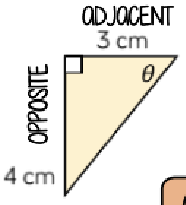

$\text{Hypotenuse}^2 = a^2 + b^2$

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

## Sin, Cos, Tan: Angles

Inverse trigonometric functions




Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\theta = \tan^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

$\theta = \sin^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \cos^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

$\tan\theta = \frac{3}{4}$

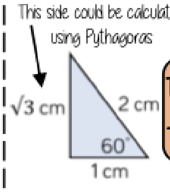
$\theta = \tan^{-1}\frac{3}{4}$

$\theta = 36.9°$

## Key angles

This side could be calculated using Pythagoras

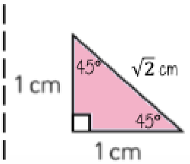
Because trig ratios remain the same for similar shapes you can generalise from the following statements




$\tan 30 = \frac{1}{\sqrt{3}}$ $\tan 60 = \sqrt{3}$

$\cos 30 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$

$\sin 30 = \frac{1}{2}$ $\sin 60 = \frac{\sqrt{3}}{2}$




$\tan 45 = 1$ $\cos 45 = \frac{1}{\sqrt{2}}$ $\sin 45 = \frac{1}{\sqrt{2}}$

## Key angles 0° and 90°

$\tan 0 = 0$ ~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle


$\sin 0 = 0$ $\sin 90 = 1$

$\cos 0 = 1$ $\cos 90 = 0$