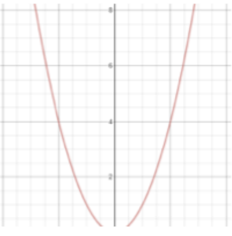
Year 9 Knowledge Organiser

Quadratic Expressions and Equations

General form of a quadratic equation

The general equation of a quadratic is $y = ax^2 + bx + c$, where a, b and c are all constant values. The +c represents the intercept and tells us where the graph will cross the y-axis.

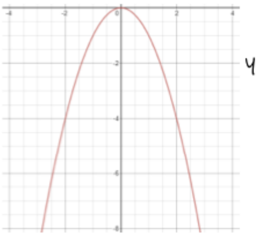
If the a is positive, the graph will form a u shape.



$y = x^2$

The graph is a smooth curve between each point and is called a parabola.

If the a is negative the graph will form a n shape.



$y = -x^2$

Expanding a linear bracket

Multiply all terms inside the bracket by the term in front of the bracket being careful with any negative numbers

e.g. $4(3a - 6) = 12a - 24$

as $4 \times 3a = 12a$ and $4 \times -6 = -24$

Expanding a double bracket

$(2x + 4)(3x + 5)$

	3x	+5
2x	$6x^2$	10x
+4	12x	20

So $6x^2 + 22x + 20$

Plotting and using quadratic graphs

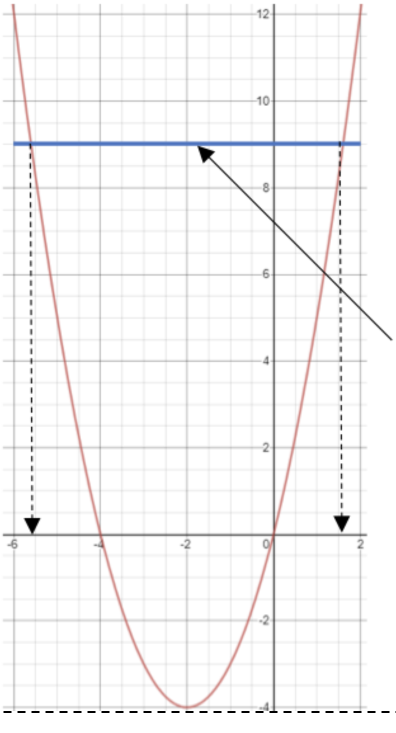
e.g. a) Complete the table of values for $y = x^2 + 4x$ and plot the graph

x	-6	-4	-2	0	2
y	12	0	-4	0	12

$y = (-6)^2 + 4 \times -6$

$y = 36 - 24 = 12$

As a quadratic graph is symmetrical, you will often see repeating values of y



b) Use the graph to find estimates for the solutions of $x^2 + 4x = 9$

We already have the graph of $y = x^2 + 4x$

We draw on to the same axis the graph of $y = 9$

Where the 2 graphs intersect (cross) we read off the two x values.

So $x = 1.5$ and $x = -5.5$

Expanding 3 brackets

First expand the first two brackets using a normal method to get a quadratic. Then use a grid to multiply the quadratic by the third bracket.

$(3x + 2)(2x - 4)(5x + 7)$

First expand $(3x + 2)(2x - 4)$

	$3x$	$+2$
$2x$	$6x^2$	$4x$
-4	$-12x$	-8

So $6x^2 - 8x - 8$

Now multiply out $(6x^2 - 8x - 8)(5x + 7)$

	$6x^2$	$-8x$	-8
$5x$	$30x^3$	$-40x^2$	$-40x$
$+7$	$42x^2$	$-56x$	-56

Diagonal boxes in the grid have similar terms which can be collected together and simplified for the final answer

So the final answer is $30x^3 + 2x^2 - 96x - 56$

Factorising Quadratics

The general form of a quadratic expression is $\mathbf{ax^2 + bx + c}$ where a, b and c are numbers.

Type 1: a = 1

When factorising a full quadratic expression, it goes into 2 brackets. The second terms in the brackets need to multiply to make the "+c" and add to make the "+b"

e.g. $x^2 + 8x + 12$

$6 \times 2 = 12$ and $6 + 2 = 8$

$(x + 6)(x + 2)$

$x^2 - 10x + 24$

$-6 \times -4 = 24$ and $-6 + -4 = -10$

$(x - 6)(x - 4)$

$x^2 - 3x + 28$

$-7 \times 4 = -28$ and $-7 + 4 = -3$

$(x - 7)(x + 4)$

Special cases: 1) No "+c" e.g. $6x^2 + 3x$ This factorises into 1 bracket rather than 2. $6x^2 + 3x = 3x(2x + 1)$

2) No "+b" and c is negative e.g. $x^2 - 25$ This is known as the **difference of two squares** and factorises into two brackets. Both brackets are the same except the sign in the middle $x^2 - 25 = (x + 5)(x - 5)$

Type 2: a > 1

This method also works for when a = 1 but takes slightly longer than just "spotting" it.

e.g. $6x^2 - 11x - 10$

Step 1 – multiply a and c together then find factors of this number which add to b

$6 \times -10 = -60$. Factors of -60 which add to -11 are -15 and +4

Step 2 – Rewrite the b term (-11x) using these two factors

$6x^2 - 15x + 4x - 10$

Step 3 – Factorise the first two terms into one bracket

$3x(2x - 5) + 4x - 10$

Step 4 – Factorise the last two terms into one bracket. Tip – it will be the same bracket as used for the first two terms

$3x(2x - 5) + 2(2x - 5)$

Step 5 – This bracket is a factor of both terms so now rewrite as two brackets

$(3x + 2)(2x - 5)$