## My mathematical journey

**What do I need to remember from before?**

Multiplication and division (NP3)

Multiplying by composing and decomposing (NP3)

Multiples and factors (NP3)

**What will I learn about in this unit?**

Repeated multiplication

Powers and roots

Prime numbers

Composing and decomposing primes

**Where does this lead?**

Order of operations (NP5)

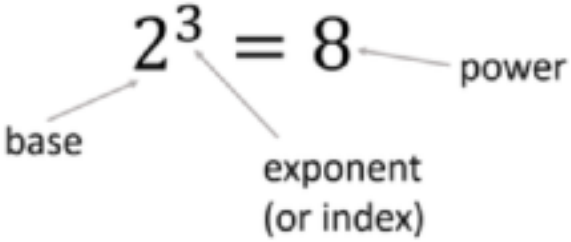
Directed numbers (NP6)

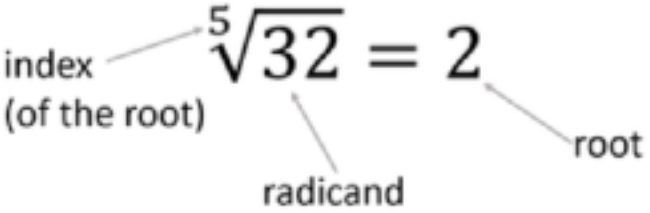
Quadratics (A11)

Index laws (NP15)

Exponential growth (NP16)

## Key words and symbols: what I need to say and write accurately




The "radical" or "root" symbol: $\sqrt{\phantom{x}}$

## Fingertip facts: what I need to learn by heart

The first fifteen square numbers:

| Square number | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th | 9th | 10th | 11th | 12th | 13th | 14th | 15th |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Value | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 | 100 | 121 | 144 | 169 | 196 | 225 |

The first ten cube numbers:

| Cube number | 1st | 2nd | 3rd | 4th | 5th | 6th | 7th | 8th | 9th | 10th |
|---|---|---|---|---|---|---|---|---|---|---|
| Value | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 | 1000 |

The prime numbers less than 100:

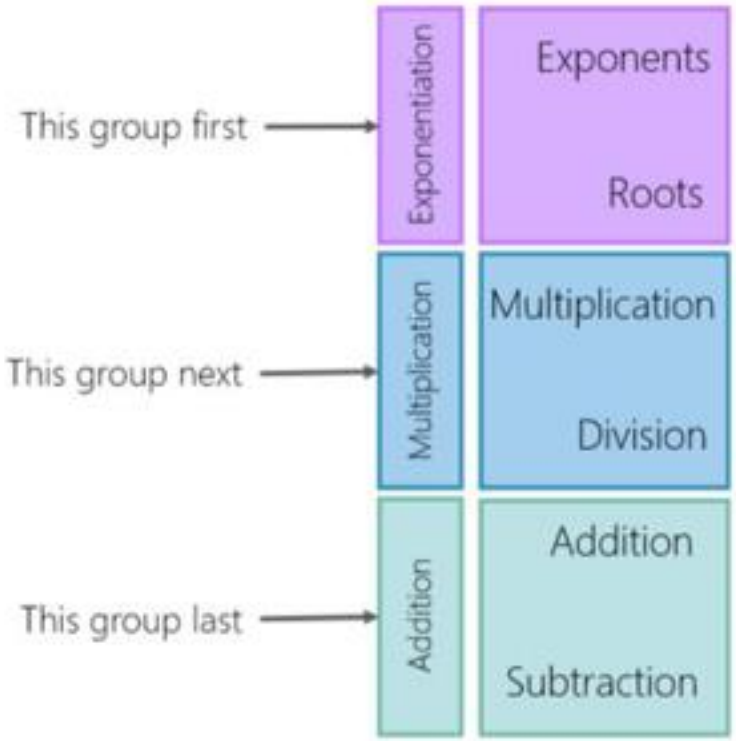
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

## My mathematical journey

| What do I need to remember from before? | What will I learn about in this unit? | Where does this lead? |
|---|---|---|
| Addition and subtraction (NP2) | Flexible calculating | Directed numbers (NP6) |
| Multiplication and division (NP3) | The order of operations | Substitution (A1, A2, A5) |
| Exponents and roots (NP4) | Using visible and invisible brackets to break the order of operations | Linear equations (A4) |

## Fingertip facts: what I need to learn by heart

The order of operations is:




To break the order, use brackets.

( )

## My mathematical journey

**What do I need to remember from before?**

Addition and subtraction with integers and decimals (NP2)

Multiplication and division with integers and decimals (NP3)

Exponents and roots (NP4)

Order of operations (NP5)

**What will I learn about in this unit?**

Direction of numbers

Using negative numbers

Calculating with negative numbers

**Where does this lead?**

Algebraic expressions (A2, A3)

Linear equations (A4)

Formulae (A5)

Graphs (A6)

Quadratic expressions (A11)

## Key words and symbols: what I need to say and write accurately

| Number | What we write | What we say |
|---|---|---|
| 6 | 6, +6, ⁺6 | "six"<br>"positive six" |
| −6 | −6, ⁻6, (−6) | "negative six" |

The sign of a number refers to whether it is positive or negative.

## Fingertip facts: what I need to learn by heart

A zero pair is a pair of positive and negative numbers of the same magnitude. Their sum is 0.

1 + -1 = 0

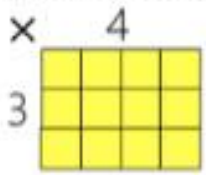
Subtraction is the same as adding the inverse. When you subtract, invert the number (flip the tile) and add.

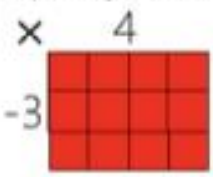
2 − 5

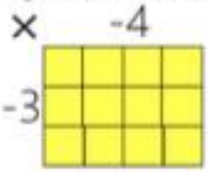
= 2 + −5

= −3

Whenever we multiply or divide by a negative number, we invert the sign (flip the tiles).



## My mathematical journey

**What do I need to remember from before?**

Addition and subtraction (NP2)

Multiplication and division (NP3)

Exponents and roots (NP4)

Order of operations (NP5)

Directed numbers (NP6)

**What will I learn about in this unit?**

Representing fractions with pictures and numerals

Calculating with fractions

Finding fractions and wholes

**Where does this lead?**

Percentages, decimals and fractions (NP8)

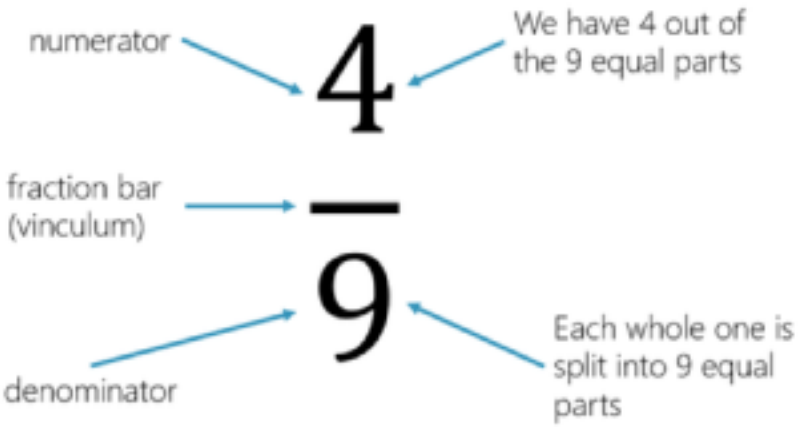
Proportional reasoning (NP10)

Ratio (NP11)

Linear equations (A4)

Algebraic fractions (A17)

## Key words and symbols: what I need to say and write accurately




| Word | Explanation |
|---|---|
| **proper fraction** | a number less than 1, written as a fraction where the numerator is less than the denominator.<br>e.g. $\frac{4}{9}$ |
| **improper fraction** | a number greater than 1, written as a fraction where the numerator is greater than the denominator.<br>e.g. $\frac{14}{9}$ |
| **mixed number** | a number greater than 1, written as an integer and a proper fraction.<br>e.g. $1\frac{5}{9}$ |

## My mathematical journey

**What do I need to remember from before?**

Number lines (NP1, 2, 3, and 6)

Decimals (NP1, 2, and 3)

Fractions (NP7)

Finding a fraction of a number (NP7)

**What will I learn about in this unit?**

Equivalent fractions, decimals and percentages

Terminating and recurring decimals

Working with percentages

**Where does this lead?**

Proportional reasoning (NP10)

Contextual graphs (A9)

Percentage change (NP10, NP13)

Recurring decimals to fractions (NP14)

## Key words and symbols: what I need to say and write accurately

$32\%$ means $\frac{32}{100}$

"percent" means "out of 100"

A **terminating decimal** has a finite (fixed) number of decimal places,
e.g. $0.215$
e.g. $0.3$

A **recurring decimal** has an infinite number of decimal places and its digits have a repeating pattern. The **repetend** is the repeating part. We use dots to show the start and end of the repetend.
e.g. $0.333333333\ldots = 0.\dot{3}$
e.g. $0.804804804\ldots = 0.\dot{8}0\dot{4}$

## Fingertip facts: what I need to learn by heart

Tenths and fifths:

| Fraction | $\frac{1}{10}$ | $\frac{2}{10}=\frac{1}{5}$ | $\frac{3}{10}$ | $\frac{4}{10}=\frac{2}{5}$ | $\frac{5}{10}=\frac{1}{2}$ | $\frac{6}{10}=\frac{3}{5}$ | $\frac{7}{10}$ | $\frac{8}{10}=\frac{4}{5}$ | $\frac{9}{10}$ | $\frac{10}{10}=1$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Decimal | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
| Percentage | 10% | 20% | 30% | 40% | 50% | 60% | 70% | 80% | 90% | 100% |

Eighths and quarters:

| Fraction | $\frac{1}{8}$ | $\frac{2}{8}=\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{4}{8}=\frac{2}{4}=\frac{1}{2}$ | $\frac{5}{8}$ | $\frac{6}{8}=\frac{3}{4}$ | $\frac{7}{8}$ | $\frac{8}{8}=\frac{4}{4}=\frac{2}{2}=1$ |
|---|---|---|---|---|---|---|---|---|
| Decimal | 0.125 | 0.25 | 0.375 | 0.5 | 0.625 | 0.75 | 0.875 | 1 |
| Percentage | 12.5% | 25% | 37.5% | 50% | 62.5% | 75% | 87.5% | 100% |

Ninths and thirds:

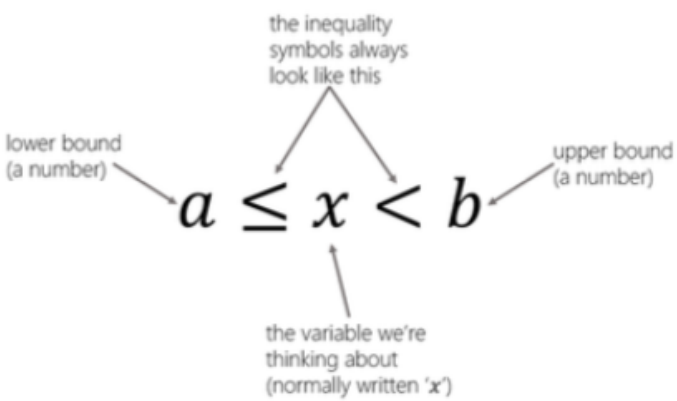
| Fraction | $\frac{1}{9}$ | $\frac{2}{9}$ | $\frac{3}{9}=\frac{1}{3}$ | $\frac{4}{9}$ | $\frac{5}{9}$ | $\frac{6}{9}=\frac{2}{3}$ | $\frac{7}{9}$ | $\frac{8}{9}$ | $\frac{9}{9}=1$ |
|---|---|---|---|---|---|---|---|---|---|
| Decimal | $0.\dot{1}$ | $0.\dot{2}$ | $0.\dot{3}$ | $0.\dot{4}$ | $0.\dot{5}$ | $0.\dot{6}$ | $0.\dot{7}$ | $0.\dot{8}$ | $0.\dot{9}=1$ |
| Percentage | $11.\dot{1}\%$ | $22.\dot{2}\%$ | $33.\dot{3}\%$ | $44.\dot{4}\%$ | $55.\dot{5}\%$ | $66.\dot{6}\%$ | $77.\dot{7}\%$ | $88.\dot{8}\%$ | $99.\dot{9}\%=100\%$ |

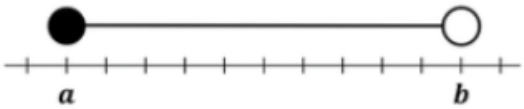
## My mathematical journey

| What do I need to remember from before? | What will I learn about in this unit? | Where does this lead? |
|---|---|---|
| Number lines: single and double (NP1 – NP8) | Using my calculator accurately and efficiently | Solving complex problems using the calculator (all future units) |
| Approximating numbers (NP1 – NP7) | Approximating numbers | Checking answers by estimating (all future units) |
| Inequalities (NP1) | Estimating answers to calculations | Problems with bounds (NP14) |
| Fractions (NP7) | Error intervals for rounding | |
| Directed numbers (NP6) | Truncation | |

## Key words and symbols: what I need to say and write accurately

- An **error interval** uses inequalities to show the range of values a number could be. We can show it with inequalities *and* on a number line.




- A **surd** is a root that does not have an integer or fraction answer, such as $\sqrt{2}$ or $\sqrt[3]{10}$.

| Symbol | ≈ | < | ≤ | > | ≥ |
|---|---|---|---|---|---|
| How to read it | is approximately equal to | is less than | is less than or equal to | is greater than | is greater than or equal to |

## Fingertip facts: what I need to learn by heart

**Time frame conversions**

| | |
|---|---|
| 1 minute | = 60 seconds |
| 1 hour | = 60 minutes |
| 1 day | = 24 hours |
| 1 week | = 7 days |
| 1 year | = 52 weeks |
| 1 year | = 365 days |
| 1 leap year | = 366 days |

**Days in the months**

| | |
|---|---|
| January: | 31 days |
| February: | 28 days (and 29 days in a leap year) |
| March: | 31 days |
| April: | 30 days |
| May: | 31 days |
| June: | 30 days |
| July: | 31 days |
| August: | 31 days |
| September: | 30 days |
| October: | 31 days |
| November: | 30 days |
| December: | 31 days |

## My mathematical journey

**What do I need to remember from before?**

Area models for multiplication (NP3)

Collecting like terms (A2)

**What will I learn about in this unit?**

Expanding expressions with brackets

Factorising expressions as the opposite of expanding

Expanding two brackets

**Where does this lead?**

Solving equations (A4)

Formulae (A5)

Inequalities (A8)

Quadratic expressions (A11)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **variable** | a number that can change its value, represented by a letter such as $x$ or a green tile |
| **constant** | a number that does not change, is fixed |
| **operation** | something that takes input numbers and turns them into output numbers, such as addition (including subtraction), multiplication (including division), exponentiation (including roots) |
| **expression** | a collection of constants, variables and operations<br>e.g. $4x$, $2p - 5$ and $x^2 + 3x + 6$ are all expressions |
| **term** | the parts of an expression separated by $+$ or $-$.<br>e.g. in the expression $4x - \frac{1}{2}y$, the terms are $4x$ and $\frac{1}{2}y$ |
| **expand** | write an expression containing brackets *without* the brackets, by multiplying<br>e.g. $2(x - 5) = 2x - 10$ |
| **factorise** | write an expression without brackets as a multiplication *with* brackets<br>e.g. $2x - 10 = 2(x - 5)$ |

## My mathematical journey

**What do I need to remember from before?**

Equality & inverse operations (NP2, NP3, NP4)

Solving equations (A1)

Simplifying expressions (A2)

Expanding brackets (A3)

**What will I learn about in this unit?**

Mathematical equality

Balancing an equation

Solving all types of linear equations

**Where does this lead?**

Rearranging formulae (A5)

Equations of a line (A6)

Quadratic equations (A12)

Using equations to solve geometry and probability problems (GM2 – GM11, SP7)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **unknown** | a number that we do not know, represented by a letter |
| **expression** | a collection of constants, variables and operations<br>e.g. $4x$, $2p - 5$ and $x^2 + 3x + 6$ are all expressions |
| **equation** | when we write two expressions equal to one another<br>e.g. $2 + 3 = 5$, $2x + 3 = 5$ and $2x + 3 = 5x - 6$ are all equations |
| **term** | the parts of an expression separated by $+$ or $-$<br>e.g. in the expression $4x - \frac{1}{2}y$, the terms are $4x$ and $\frac{1}{2}y$ |
| **solve** | when we solve an equation, we find out what the value of the unknown is |

## Fingertip facts: what I need to learn by heart

An equation must always be balanced: whatever we do to one side we must also do to the other.

## My mathematical journey

**What do I need to remember from before?**

Arithmetic strategies (NP1, 2, 3, 4)

Order of operations (NP5)

Negative numbers (NP6)

Algebraic expressions (A1, 2, 3)

Solving equations (A4)

**What will I learn about in this unit?**

Substituting numbers into expressions and formulae

Writing and using formulae

Rearranging formulae to change the subject

**Where does this lead?**

All further algebra units

Using formulae in geometry (GM3 onwards)

Advanced proportion (NP13)

Advanced probability and statistics (A Level)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **variable** | A letter that represents many numbers (a letter whose value can vary)<br><br>e.g. $x$, $y$, $\theta$ |
| **constant** | A fixed number<br><br>e.g. $2$, $-1.8$, $\pi$ |
| **expression** | A collection of any variables, constants and operations<br><br>e.g. $2x + 5$, $a - b$, $3p$, $\frac{n+5}{7}$, $3 \times 4 - 2^3$ |
| **substitute** | Replace a variable with a constant<br><br>e.g. When $x = 3$, the value of $2x - 1$ is $2 \times 3 - 1$, or 5. |
| **evaluate** | Work out the value of a calculation.<br><br>e.g. "Evaluate 2 + 3" means "Work out the value of 2 + 3." |
| **formula (pl. formulae or formulas)** | A set of instructions to work something out.<br>A formula can be written in words, as an expression or as an equation.<br><br>Here is a formula in words: Area = length × width<br>Here is the formula as an expression: $lw$<br>Here is the formula as an equation: $A = lw$ |
| **subject** | The subject of a formula is the variable that is 'on its own' on one side of the equation.<br><br>e.g. In the formula $A = lw$, the subject is $A$. |
| **rearrange** | Rearranging a formula means changing its subject.<br><br>e.g. $A = lw$ can be rearranged to make $l$ the subject: $l = \frac{A}{w}$ |

## My mathematical journey

**What do I need to remember from before?**

Number lines (NP1, 2, 3, and 6)

Substitution (A1 and A5)

Writing expressions, equations and formulae (A2, A3, A4 and A5)

**What will I learn about in this unit?**

Plotting and using coordinates

The links between graphical and algebraic representations of equations

Gradient as a measure of steepness

**Where does this lead?**

Sequences (A7)

Inequalities on graphs (A8, A10)

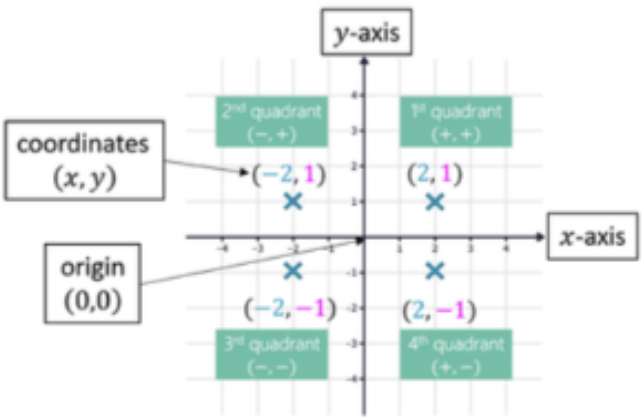
Advanced equations of lines (A10)

Quadratic graphs (A12)

Advanced graphs (A14, A15)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **midpoint** | the point exactly in the middle of two others |
| **gradient** | the steepness of a line |
| **$y$-intercept** | where a graph crosses the $y$-axis |
| **$x$-intercept** | where a graph crosses the $x$-axis |
| **parallel** | describing two lines that have the same gradient so will never intersect |
| **to intersect** | to cross – we say two lines intersect |
| **simultaneously** | at the same time |
| **parabola** | the name of the shape of a quadratic graph |
| **vertex** | the turning point of a quadratic graph |




## Fingertip facts: what I need to learn by heart

The equation of any straight line can be written in the form $y = mx + c$.

The coefficient of $x$ gives the gradient

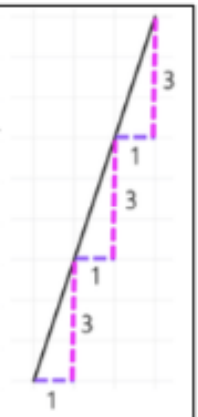
$$y = mx + c$$

The constant gives the $y$ intercept

The steepness of a line is its gradient.

The value of the gradient is the number of units moved vertically for every unit moved horizontally. In other words: 1 right, ____ up/down.

This line goes 1 right, 3 up, so its gradient is 3.



## My mathematical journey

**What do I need to remember from before?**

Arithmetic (NP2, NP3, NP4)

Solving linear equations (A4)

Formulae, including substitution (A5)

Linear graphs (A6)

**What will I learn about in this unit?**

Linear sequences

$n$th term formulae for linear sequences

Recognising non-linear sequences

**Where does this lead?**

Advanced linear graphs (A10)

Advanced sequences – quadratic and geometric (A13)

Recurrence relations (A13)

Exponential functions (A15)

Sequences on A-Level maths

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **term** | A number in a sequence. Terms have position 1, 2, 3, 4, and so on, and these positions are labelled with the variable $n$.<br>e.g. in the sequence 5, 7, 9, 11, ... the 1st term (where $n = 1$) is 5 and the 4th term (where $n = 4$) is 11. |
| **term-to-term rule** | We can define a sequence with a term-to-term rule, which tells us where to start and how to get from one term to the next.<br>e.g. in the sequence 5, 7, 9, 11, ... the term-to-term rule would be 'start at 5 and add 2 every time' |
| **increasing sequence** | A sequence where each term is greater than the one before.<br>e.g. 5, 7, 9, 11, ... |
| **decreasing sequence** | A sequence where each term is less than the one before.<br>e.g. 11, 9, 7, 5, ... |
| **$n$th term formula** | A formula that calculates the value of each term, using its position, $n$. For this reason it is sometimes called the position-to-term formula.<br>e.g. For the sequence 5, 7, 9, 11, ... the $n$th term formula is $2n + 3$ |
| **coefficient** | A number/letter that multiplies another in an expression.<br>e.g. In the expression $2n + 3$, the coefficient of $n$ is 2 and the coefficient of 2 is $n$. |
| **linear sequence** | A sequence where the difference between terms is constant (doesn't change).<br>e.g. 5, 7, 9, 11, ... (the difference is 2) or 10, 7, 4, 1, ... (the difference is $-3$). |
| **quadratic sequence** | A sequence where the differences between terms form a linear sequence.<br>e.g. 1, 4, 9, 16, 25, ... (the differences are 3, 5, 7, 9, ..., which is itself a linear sequence). |
| **geometric sequence** | A sequence where there is a constant multiplier between terms.<br>e.g. 1, 2, 4, 8, 16, ... (each term is multiplied by 2 to get the next) |
| **Fibonacci-style sequence** | A sequence where each term is the sum of the previous two.<br>e.g. 1, 4, 5, 9, 14, 23, ... |

## Fingertip facts: what I need to learn by heart

*The sequence of square numbers*: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

*The sequence of cube numbers*: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, ...

*The triangular (or triangle) numbers*: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

*The Fibonacci sequence*: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

## My mathematical journey

**What do I need to remember from before?**

Inequality symbols, < and > (KS2)

Solving linear equations (A4)

Plotting vertical and horizontal graphs (A6)

**What will I learn about in this unit?**

Reading, writing and interpreting inequalities

Solving linear inequalities, including in contexts

Plotting simple inequalities in 2D

**Where does this lead?**

Inequalities in 2D (A??)

Quadratic inequalities (A??)

Non-linear inequalities (A Level Maths)

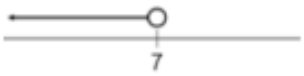
Linear programming (A Level Further Maths)

## Key words and symbols: what I need to say and write accurately

| Word or symbol | Explanation |
|---|---|
| > | is greater than |
| < | is less than |
| ≥ | is greater than or equal to |
| ≤ | is less than or equal to |
| equation | a statement that two quantities have equal value,<br>e.g. 5 + 2 = 10 – 3 |
| Inequality | a statement that two quantities do not have equal value,<br>e.g. 5 + 2 < 12 + 1 |
| comparative inequality | an inequality that compares two values,<br>e.g. 4 > 1 or $x > 8$ or $1 \leq x$ |
| restrictive inequality | a 'double' inequality that puts an upper and lower limit on a number,<br>e.g. $5 \leq x < 10$ |

Phrases meaning 'less than'

$x < 7$




*any number which is...*

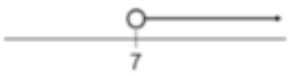
less than 7

up to (but not including) 7

up to (and excluding) 7

Phrases meaning 'greater than'

$x > 7$




*any number which is...*

greater than 7

exceeding 7

Phrases meaning 'less than or equal to'

$x \leq 7$


*any number which is...*

less than or equal to 7

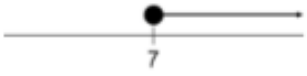
at most 7

no greater/more than 7

up to (and including) 7

Phrases meaning 'greater than or equal to'

$x \geq 7$




*any number which is...*

greater than or equal to 7

at least 7

no less than 7

## Fingertip facts: what I need to learn by heart

Inequalities can be read in both directions.
These two statements mean the same thing.

$5 > 3$ $\qquad$ $3 < 5$

*read this way* → *5 is greater than 3* $\qquad$ *read this way* → *5 is greater than 3*

*read this way* → *3 is less than 5* $\qquad$ *read this way* → *3 is less than 5*

If we multiply or divide an inequality by a negative number, the direction of the sign reverses due to the rotating effect of multiplication by negatives.

If $-x > 2$,
then $x < -2$

## My mathematical journey

| What do I need to remember from before? | What will I learn about in this unit? | Where does this lead? |
|---|---|---|
| Multiplicative reasoning (NP3) | Direct and inverse proportion | Ratio (NP11) |
| Fractions (NP7) | Proportional reasoning in various contexts | Advanced proportion and rates of change (NP13) |
| Double number lines and ratio tables (NP8) | Percentage changes and decimal multipliers | Contextual graphs (A9) |
| Percentages (NP8) | | Probability (SP3) |

## Key words and symbols: what I need to say and write accurately

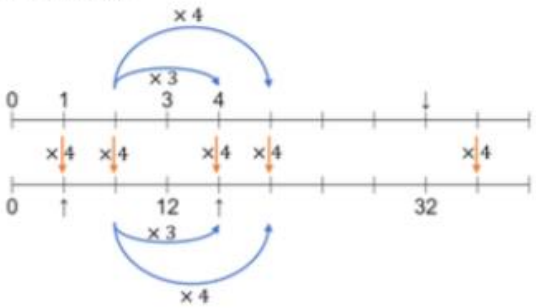
If two quantities are in **direct proportion**, the following two facts are true:

- There is a multiplicative relationship between them (e.g. if one doubles, the other doubles).
- If one is 0, the other is 0.

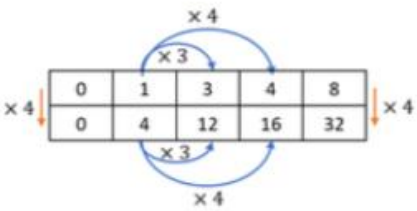
If two quantities are in **inverse proportion**, the following fact is true:

- There is an inverse multiplicative relationship between them (e.g. if one doubles, the other halves).

A **double number line** shows a multiplicative relationship.




A **ratio table** shows a multiplicative relationship, like a double number line but without the scale.




*(Notice how both these diagrams show the same information.)*

## Fingertip facts: what I need to learn by heart

- When working with direct or inverse proportion, I can only multiply or divide.
- To increase a quantity by a percentage, I add the percentage onto 100%, convert this to a decimal and multiply.
  - e.g. To increase £40 by 12%, I find 100% + 12% = 112% = 1.12 and calculate £40 × 1.12
- To decrease a quantity by a percentage, I subtract the percentage from 100%, convert this to a decimal and multiply.
  - e.g. To decrease £40 by 12%, I find 100% − 12% = 88% = 0.88 and calculate £40 × 0.88

## My mathematical journey

**What do I need to remember from before?**

Multiplication and division; multiples and factors (NP3)

Writing values as a fraction; equivalent fractions (NP7)

Ratio tables (NP10)

**What will I learn about in this unit?**

Using ratio notation

Equivalent ratios and simplifying

Ratios and fractions

Finding values from parts or the whole

**Where does this lead?**

Combining ratios (NP13)

Similar area and volume (GM8)

Geometric sequences (A13)

Advanced ratio (NP16)

Vectors (GM10)

## Key words and symbols: what I need to say and write accurately

A **ratio** describes the **multiplicative relationship** between two quantities.

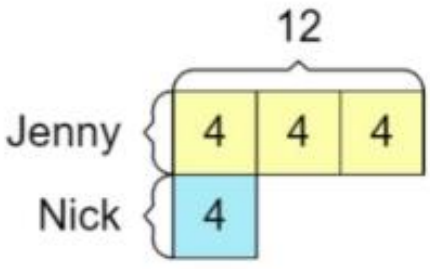
2 : 3

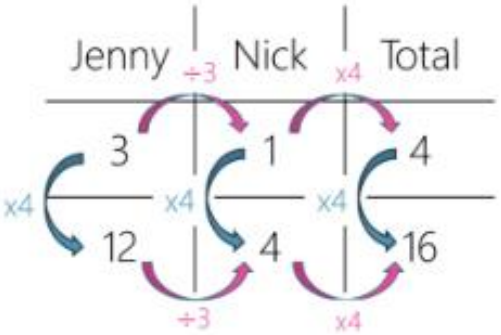
shaded parts : proportional to unshaded parts

We use a **colon** **:** to separate parts of a ratio.

## Key representations

We can use **bar models** and **ratio tables** to help us solve ratio problems. These two diagrams represent the same situation.



## My mathematical journey

**What do I need to remember from before?**

Lines and angles (KS2)

Measuring (KS2)

**What will I learn about in this unit?**

Labelling lines and angles

Drawing and measuring lines and angles

Using compasses and a protractor

Constructions and loci

**Where does this lead?**

Polygons and angles (GM2)

Congruence and similarity (GM4)

Advanced drawing, measuring and constructing (GM7)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **point** | A point has no length or width (it exists in no dimensions, or 0D) |
| **line** | A line has infinite length and no width (it exists in one dimension, or 1D).<br>We use arrows to show its infinity in both directions. |
| **ray** | A ray is a section of a line with a starting point that continues infinitely in one direction.<br>We use an arrow to show its infinity in one direction. |
| **line segment** | A line segment is a section of a line with a starting point and an end point. |
| **construct** | We construct when we only uses our compasses and straight edge (like a ruler). |
| **bisector** | 'Bisect' means 'cut in half'. A bisector is a line that cuts another in half. |
| **perpendicular** | Perpendicular lines meet at a right angle. |
| **equidistant** | Equidistant means an equal distance from two points or lines. |
| **locus (pl. loci)** | The path of all points that fit a condition. |

Angle types:

| Acute | Right | Obtuse | Straight | Reflex | Full turn |
|---|---|---|---|---|---|
| $0° < \theta < 90°$ | $90° = \theta$ | $90° < \theta < 180°$ | $180° = \theta$ | $180° < \theta < 360°$ | $360° = \theta$ |

Greek letters:

$\alpha$ (alpha) $\beta$ (beta) $\gamma$ (gamma) $\theta$ (theta)

## Fingertip facts: what I need to learn by heart

You will need to learn the constructions for:

1. a perpendicular bisector
2. an angle bisector
3. a perpendicular from a point on a line
4. a perpendicular from a point near a line

## My mathematical journey

**What do I need to remember from before?**

Measuring and drawing angles (Key Stage 2, GM1)

Basic angle facts (NP2)

**What will I learn about in this unit?**

Angle facts about lines and polygons

Types of quadrilaterals and other polygons

Bearings

**Where does this lead?**

Congruence and similarity (GM4)

Trigonometry (GM5, GM9)

Solving geometric problems, including circle theorems (GM6, GM7, GM11)

## Key words and symbols: what I need to say and write accurately

A vertex (plural, vertices) is made when two lines meet. Sometimes called a corner.

***Lines:*** vertical, horizontal, parallel, perpendicular, oblique

***Angles***: acute, obtuse, reflex, alternate, corresponding, interior

***Triangles***: scalene, isosceles, equilateral

***Quadrilaterals***: square, rectangle, parallelogram, rhombus, (isosceles) trapezium, kite, arrowhead

***Polygons***: triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon

***Symmetry*** can be reflective or rotational

## Fingertip facts: what I need to learn by heart

| Polygon | Number of sides | Interior angle sum |
|---|---|---|
| Triangle | 3 | 180° |
| Quadrilateral | 4 | 360° |
| Pentagon | 5 | 540° |
| Hexagon | 6 | 720° |
| Heptagon | 7 | 900° |
| Octagon | 8 | 1080° |
| Nonagon | 9 | 1260° |
| Decagon | 10 | 1440° |

Notice that the interior angle sum increases by 180° each time.

**Angle facts**

- Adjacent angles on a straight line sum to 180°.
- Angles around a point sum to 360°.
- Vertically opposite angles are equal.
- Angles in parallel lines on adjacent or corresponding sides of the transversal are equal.
- Three-figure bearings are measured clockwise starting from north.

## My mathematical journey

**What do I need to remember from before?**

Area of a rectangle (KS2)

Area of rectilinear shapes (NP3)

Perimeter (KS2 & NP2)

Types of polygons (GM2)

**What will I learn about in this unit?**

Units of length and of area

Area of a triangle

Area of quadrilaterals

Area of a circle

**Where does this lead?**

Area and circumference of a circle (GM6)

Surface area and volume (GM8)

Advanced area (GM9)

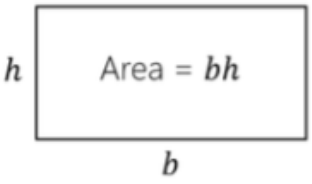
Solving geometric problems (GM11)

## Key words and symbols: what I need to say and write accurately

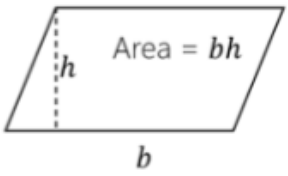
| Word | Explanation |
|---|---|
| **Area** | A measure of the two-dimensional space inside a shape. |
| **Perimeter** | A measure of the one-dimensional boundary that creates a shape. |
| **Perpendicular height** | The height of a shape which is at a right angle to its base. |
| **Radius** | The length from the centre of a circle to its edge. |
| **Diameter** | The length straight across the centre of a circle from edge to edge. Double the radius. |

## Fingertip facts: what I need to learn by heart

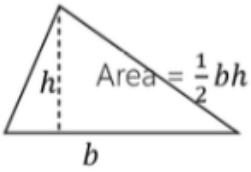
Area of a rectangle = base × perpendicular height



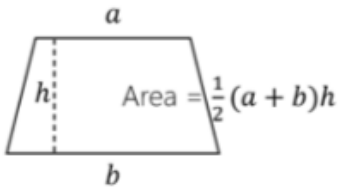

Area of a parallelogram = base × perpendicular height




Area of a triangle = $\frac{1}{2}$ × base × perpendicular height




Area of a trapezium = $\frac{1}{2}$ × sum of the parallel sides × perpendicular height




The area of other polygons can be found by splitting them into triangles.

Area of a circle = pi × square radius

## My mathematical journey

**What do I need to remember from before?**

Place value, multiplication and division by powers of 10 (NP1)

Operations (NP2, NP3, NP4)

Laws of indices (NP5)

**What will I learn about in this unit?**

Writing large and small numbers in standard form.

Calculating with numbers in standard form.

**Where does this lead?**

Compound units (NP13)

Index laws (NP15)

Exponential growth (NP16)

You will apply this knowledge in Physics, Chemistry and Biology

## Key words and symbols: what I need to say and write accurately

Numbers in standard form look like this.

$$x \times 10^{n}$$

$1 \leq x < 10$

$n$ is an integer

The power of 10 tells us how many place value columns up or down to move.

e.g. $123\,000 = \mathbf{1.23 \times 10^{5}}$

$0.000\,012\,3 = \mathbf{1.23 \times 10^{-5}}$

## Fingertip facts: what I need to learn by heart

**Prefixes for SI units**

| Standard form | Amount | Prefix | Symbol |
|---|---|---|---|
| $1 \times 10^{12}$ | 1 000 000 000 000 | tera | T |
| $1 \times 10^{9}$ | 1 000 000 000 | giga | G |
| $1 \times 10^{6}$ | 1 000 000 | mega | M |
| $1 \times 10^{3}$ | 1 000 | kilo | k |
| $1 \times 10^{2}$ | 100 | hecto | h |
| $1 \times 10^{1}$ | 10 | deca | da |
| $1 \times 10^{0}$ | 1 | | |
| $1 \times 10^{-1}$ | 0.1 | deci | d |
| $1 \times 10^{-2}$ | 0.01 | centi | c |
| $1 \times 10^{-3}$ | 0.001 | milli | m |
| $1 \times 10^{-6}$ | 0.000001 | micro | μ |
| $1 \times 10^{-9}$ | 0.000000001 | nano | n |
| $1 \times 10^{-12}$ | 0.000000000001 | pico | p |

## My mathematical journey

**What do I need to remember from before?**

Percentage multipliers (NP8, NP10)

Proportional reasoning (NP10)

Cartesian graphs (A6, A9, A10)

Manipulating algebra (A5, A10)

**What will I learn about in this unit?**

Reverse percentages

Simple interest

Direct and inverse proportion

Compound units

Combining ratios

**Where does this lead?**

Compound interest (NP16)

Proportion with squares, cubes and roots (NP16)

Instantaneous and average rates of change (A16)

Areas under graphs (A16)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **p.a.** | *per annum*, Latin for *"every year"* |
| **direct proportion** | as one amount increases, the other increases at the same rate |
| $k$ | the constant of proportionality, which is the rate of change |
| $\propto$ | *is proportional to* |
| **inverse proportion** | as one amount increases, the other *decreases* at the same rate |
| **density** | a measure of how spread out particles in an object are |
| **pressure** | a measure of how spread out a force is |
| **speed** | a measure of distance per unit of time |
| **compound units** | a measure combining two other measures |

## Fingertip facts: what I need to learn by heart

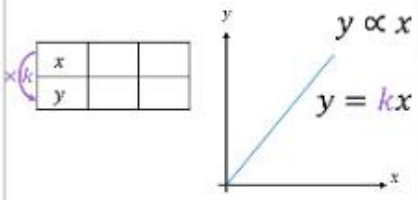
To calculate a percentage: $\textbf{original} \times \textbf{multiplier} = \textbf{new}$

To return to the original whole: $\textbf{original} = \frac{\textbf{new}}{\textbf{multiplier}}$

### Direct Proportion

As one increases, the other increases at the same rate.

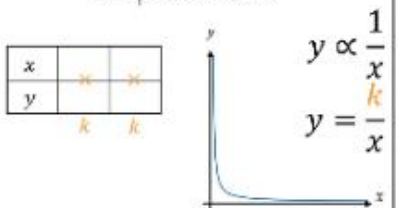
$y$ is proportional to $x$




### Inverse Proportion

As one increases, the other decreases at the same rate.

$y$ is proportional to the reciprocal of $x$




$$\text{Density} = \frac{\text{mass}}{\text{volume}} \qquad \text{Pressure} = \frac{\text{force}}{\text{area}} \qquad \text{Speed} = \frac{\text{distance}}{\text{time}}$$

## My mathematical journey

**What do I need to remember from before?**

Multiplying to scale (NP3, NP10)

Reflective and rotational symmetry (GM2)

Properties of shapes (GM2)

**What will I learn about in this unit?**

Congruence and similarity

Congruent transformations: translation, reflection, rotation

Similar transformations: enlargement

**Where does this lead?**

Trigonometry (GM5)

Area and volume in similar solids (GM8)

Problems with congruence and similarity (GM11)

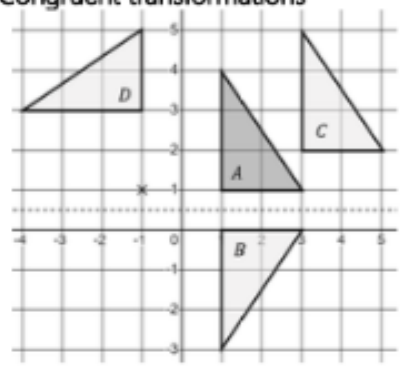
Transforming graphs (A15)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| congruent, ≅ | identical in size and shape, but not necessarily orientation or direction |
| transformation | a mathematical change, using translation, rotation, reflection or enlargement |
| image | a shape *after* a transformation has happened |
| vector | a mathematical object that tells you how far to move and in what direction<br>it can be shown with an arrow or with column notation<br><br>e.g. this arrow and column vector both communicate "one left, two down"<br>$\begin{pmatrix}-1\\-2\end{pmatrix}$  |
| similar | same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio) |

## Fingertip facts: what I need to learn by heart

**Congruent transformations**




**A** to **B** is a reflection in the line $y = \frac{1}{2}$.

**A** to **C** is a translation by the vector $\begin{pmatrix}2\\1\end{pmatrix}$

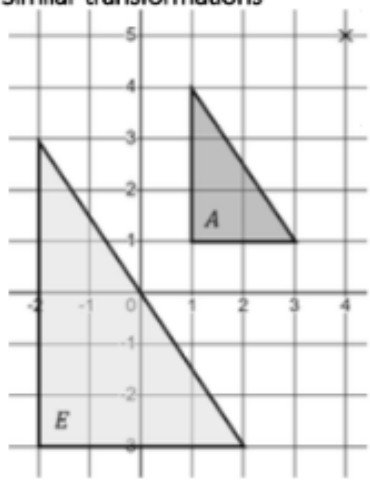
**A** to **D** is a rotation 90° anticlockwise around the centre $(-1, 1)$.

Reflections need a mirror line.

Translations need a vector.

Rotations need an angle, direction and centre.

**Similar transformations**




**A** to **E** is an enlargement of scale factor 2 from $(4, 5)$.

Enlargements need a scale factor and centre.

## My mathematical journey

**What do I need to remember from before?**

Plotting graphs (A6)

$y = mx + c$ (A6)

Finding gradient (A6)

Ratio tables and direct proportion (NP10, NP11)

**What will I learn about in this unit?**

Reading, drawing and interpreting graphs used in various contexts

Finding speed

Distance-time and speed-time graphs

**Where does this lead?**

Compound units: speed, density, pressure and more (NP13)

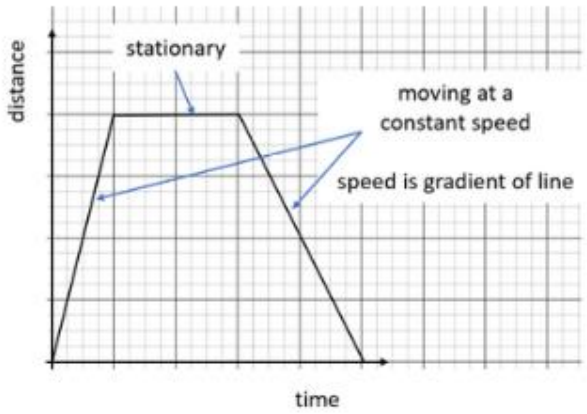
Gradient of and area under of non-linear graphs (A16)

## Key words and symbols: what I need to say and write accurately

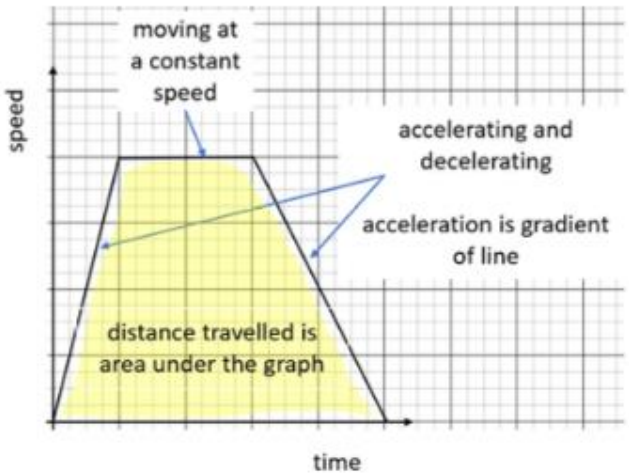
| Word | Explanation |
|---|---|
| **gradient** | the steepness of a line: for every one unit right, it is the number of units up/down.<br><br>On a contextual graph, the gradient represents the change in the vertical quantity *per one unit* of the horizontal quantity<br>e.g. the exchange rate between two currencies, or the cost per unit time. |
| **$y$-intercept** | where a graph crosses the $y$-axis<br><br>On a contextual graph, the $y$-intercept represents the value of the vertical quantity when the horizontal quantity is 0<br>e.g. a fixed or standing charge |
| **speed** | a quantity that combines distance and time, working out the distance travelled *per one unit* of time.<br>e.g. miles in 1 hour (miles per hour, mph), or metres in 1 second (metres per second, m/s) |

## Fingertip facts: what I need to learn by heart

A distance-time graph




A speed-time graph



## My mathematical journey

**What do I need to remember from before?**

Solving linear equations (A4)

Rearranging formulae (A5)

$y = mx + c$ (A6)

Linear inequalities (A8)

**What will I learn about in this unit?**

Solving problems with graphs

Finding lines parallel and perpendicular to others

Solving simultaneous equations

Plotting inequalities in two dimensions

**Where does this lead?**

Quadratic graphs (A12)

Non-linear simultaneous equations, non-linear inequalities (A14)

Tangents and normal (A15)

Gradient of a curve (A16)

Using graphs to represent complex problems (A Level Maths)

## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **$y$-intercept** | where a graph crosses the $y$-axis |
| **$x$-intercept** or **root** | where a graph crosses the $x$-axis |
| **satisfy** | a number satisfies an equation when it solves the equation |
| **gradient** | the steepness of a line |
| **parallel** | describing two lines that have the same gradient, so will never intersect |
| **perpendicular** | describing two lines that meet at right angles to each other |
| **to intersect** | to cross – we say two lines intersect |
| **simultaneously** | at the same time |
| $\parallel$ | *is parallel to* |
| $\perp$ | *is perpendicular to* |
| **region** | an area on a graph |
| **boundary** | a line that marks the edge of a region |

## Fingertip facts: what I need to learn by heart

If a point $(x, y)$ lies on a line, its coordinates can be substituted for $y$ and $x$ in the equation of the line and they will satisfy the equation.

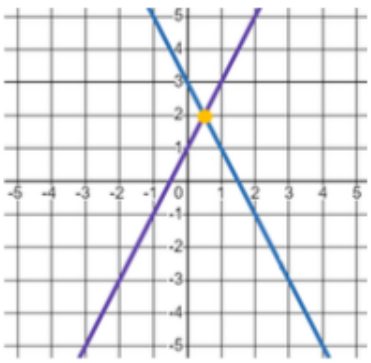
The gradient of a line is the amount up/down it moves for every one unit right. We can work this out by calculating the ratio $\frac{\text{vertical}}{\text{horizontal}}$ between two points on the line.

The gradients of parallel lines are the same.

The gradients of perpendicular lines are the negative reciprocal of each other: $+\frac{a}{b} \perp -\frac{b}{a}$

On the $y$-axis, $x = 0$. On the $x$-axis, $y = 0$.

When we solve simultaneous equations, we find the point of intersection of graphs of the two equations.



## My mathematical journey

**What do I need to remember from before?**

Number lines (NP1, 2, 3, and 6)

Decimals (NP1, 2, and 3)

Fractions (NP7)

Frequency tables (SP1)

**What will I learn about in this unit?**

Systematic listing and the product rule

Experimental and theoretical probability

Probability diagrams

**Where does this lead?**

Sets and Venn diagrams (SP5)

Sampling and data analysis (SP6)

Advanced probability problems (SP7)

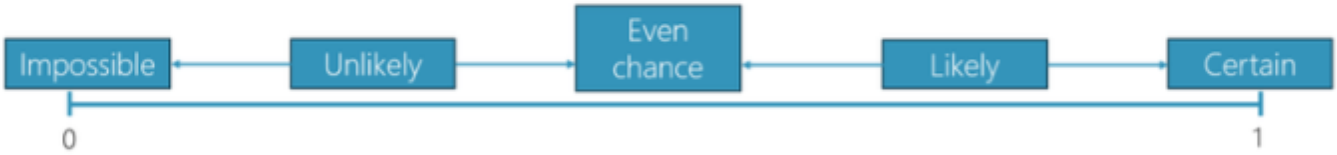
## Key words and symbols: what I need to say and write accurately

| Word | Explanation |
|---|---|
| **systematic** | working in an organised way |
| **relative frequency** | the proportion of times something happens |
| **outcome** | a result we could get from a probability experiment<br>e.g. rolling a fair six-sided die gives the outcomes 1, 2, 3, 4, 5, and 6 |
| **event** | one or more outcomes<br>e.g. rolling a square number on a die |
| **fair** | all outcomes are equally likely |
| **biased** | some outcomes are more likely than others |
| **mutually exclusive** | events which cannot happen at the same time |
| **independent** | If events are independent, they do not influence or affect each other.<br>e.g. if I flip a coin twice, the outcome of the first flip has no effect on the outcome of the second flip: the events are independent. |

## Fingertip facts: what I need to learn by heart

$$P(\text{event}) = \frac{\text{outcomes we want}}{\text{total outcomes}}$$

Probabilities can be represented by a number between 0 and 1.




The sum of all mutually exclusive events is 1.

If events $A$ and $B$ are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$

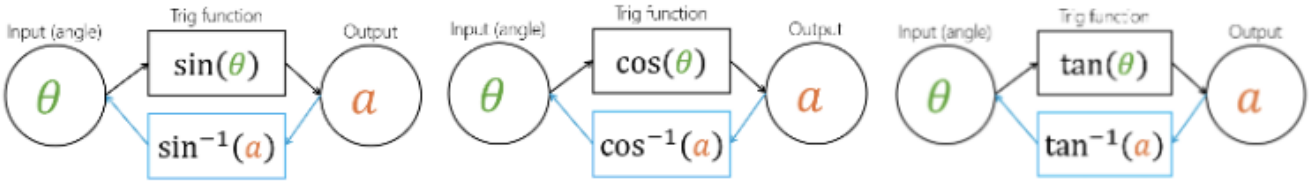
$P(\text{not } A) = 1 - P(A)$. This can also be written $P(A') = 1 - P(A)$

## My mathematical journey

| What do I need to remember from before? | What will I learn about in this unit? | Where does this lead? |
|---|---|---|
| Powers and roots (NP4) | Pythagoras' Theorem | Advanced length and area (GM9) |
| Substitution and rearranging a formulae (A5) | Trigonometry in right-angled triangles | Trigonometric graphs (A13) |
| Angles in triangles (GM2) | Exact values in surd form | A Level mathematics, physics, engineering and more |
| Similarity (GM4) | | |

## Key words and symbols: what I need to say and write accurately

The trigonometric functions have angles as their input. Their inverse functions are written using $^{-1}$.



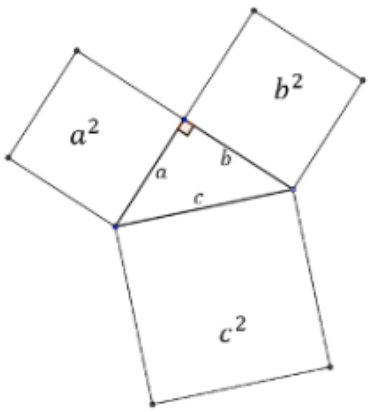

The **hypotenuse** is the longest side in a right-angled triangle, opposite the right angle.
The other two sides are called the **adjacent** and opposite. They change depending on the angle you look at.


## Fingertip facts: what I need to learn by heart

### Pythagoras' Theorem

For any right-angled triangle,

$$a^2 + b^2 = c^2.$$




### Trigonometry

Three equations:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

A mnemonic to help you remember the equations:

$$S^{O}H \quad C^{A}H \quad T^{O}A$$

The exact values of sine, cosine and tangent for key angles:

| Angle, $\theta$ | 30° | 45° | 60° |
|---|---|---|---|
| $\sin\theta$ | $\frac{\sqrt{1}}{2} = \frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ |
| $\cos\theta$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{1}}{2} = \frac{1}{2}$ |
| $\tan\theta$ | $\frac{1}{\sqrt{3}}$ | 1 | $\frac{\sqrt{3}}{1} = \sqrt{3}$ |

## My mathematical journey

**What do I need to remember from before?**

Angle facts (GM2)

Area and perimeter (GM3)

Rearranging formulae (A5)

**What will I learn about in this unit?**

Naming the parts of a circle

Finding the area and perimeter of circles and sectors of circles

Finding angles in circles using circle theorems

**Where does this lead?**

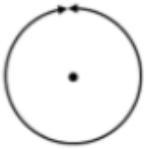
Surface area and volume (GM8)

Advanced length and area (GM9)

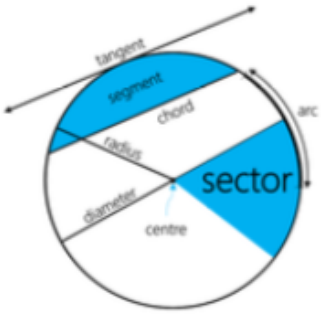
## Key words and symbols: what I need to say and write accurately

The <u>circumference</u> is the perimeter of a circle.

We use $C$ for 'circumference'.



We use $d$ for 'diameter' and $r$ for 'radius'.




<u>Semicircle</u>
A half circle

<u>Quadrant</u>
A quarter circle

<u>Concentric</u> circles share the same centre

## Fingertip facts: what I need to learn by heart

$\pi \approx 3.14$

<u>Circle formulae</u>

Circumference: $C = \pi d$ or $C = 2\pi r$

Area: $A = \pi r^2$

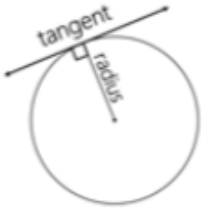
<u>Sector formulae</u>

We use $l$ for 'arc length'.

Arc length: $l = \frac{\theta}{360}\pi d$

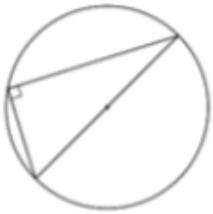
Sector area: $A = \frac{\theta}{360}\pi r^2$

<u>Circle theorems</u>




A radius and a tangent are perpendicular.

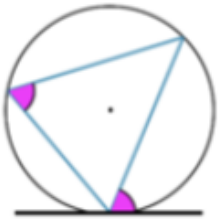

The angle at the centre is twice the angle at the circumference (subtended by the same arc).



The angle in a semicircle is 90°.


Angles at the circumference subtended by the same arc are equal. (Angles in the same segment are equal).


Opposite angles in a cyclic quadrilateral are sum to 180°.



The Alternate Segment Theorem.