# What do I need to remember from before?

Solving linear equations (A4)

Rearranging formulae (A5)

$$y = mx + c$$
 (A6)

Linear inequalities (A8)

# What will I learn about in this unit?

Solving problems with graphs

Finding lines parallel and perpendicular to others

Solving simultaneous equations

Plotting inequalities in two dimensions

## Where does this lead?

Quadratic graphs (A12)

Non-linear simultaneous equations, non-linear inequalities (A14)

Tangents and normal (A15)

Gradient of a curve (A16)

Using graphs to represent complex problems (A Level Maths)

## Key words and symbols: what I need to say and write accurately

Word	Explanation			
y-intercept	where a graph crosses the y-axis			
x-intercept or root	where a graph crosses the x-axis			
satisfy	a number satisfies an equation when it solves the equation			
gradient	the steepness of a line			
parallel	describing two lines that have the same gradient, so will never intersect			
perpendicular	describing two lines that meet at right angles to each other			
to intersect	to cross – we say two lines intersect			
simultaneously	at the same time			
ı	is parallel to			
1	is perpendicular to			
region	an area on a graph			
boundary	a line that marks the edge of a region			

# Fingertip facts: what I need to learn by heart

If a point (x, y) lies on a line, its coordinates can be substituted for y and x in the equation of the line and they will satisfy the equation.

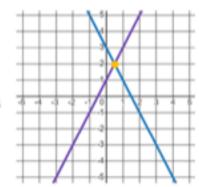
The gradient of a line is the amount up/down it moves for every one unit right. We can work this out by calculating the ratio  $\frac{\text{vertical}}{\text{horizontal}}$  between two points on the line.

The gradients of parallel lines are the same.

The gradients of perpendicular lines are the negative reciprocal of each other:  $+\frac{a}{b} + \frac{b}{a}$ 

On the y-axis, x = 0. On the x-axis, y = 0.

When we solve simultaneous equations, we find the point of intersection of graphs of the two equations.



# What do I need to remember from before?

Multiplying to scale (NP3, NP10)

Reflective and rotational symmetry (GM2)

Properties of shapes (GM2)

## What will I learn about in this unit?

Congruence and similarity

Congruent transformations: translation, reflection, rotation

Similar transformations: enlargement

### Where does this lead?

Trigonometry (GM5)

Area and volume in similar solids (GM8)

Problems with congruence and similarity (GM11)

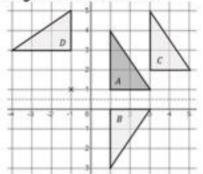
Transforming graphs (A15)

## Key words and symbols: what I need to say and write accurately

Word	Explanation
congruent, ≅	identical in size and shape, but not necessarily orientation or direction
transformation	a mathematical change, using translation, rotation, reflection or enlargement
image	a shape after a transformation has happened
vector	a mathematical object that tells you how far to move and in what direction it can be shown with an arrow or with column notation e.g. this arrow and column vector both communicate "one left, two down" $ \begin{pmatrix} -1 \\ -2 \end{pmatrix} $
similar	same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio)

# Fingertip facts: what I need to learn by heart

## Congruent transformations



A to B is a reflection in the line  $y = \frac{1}{x}$ .

A to C is a translation by the vector  $\binom{2}{1}$ 

A to D is a rotation 90° anticlockwise around the centre (-1,1).

Reflections need a mirror line.

Translations need a vector.

Rotations need an angle. direction and centre.

E A to E is an enlargement of

Similar transformations

Enlargements need a scale factor and centre.

scale factor 2 from (4,5).

## What do I need to remember from before?

Powers and roots (NP4)

Substitution and rearranging a formulae (A5)

Angles in triangles (GM2)

Similarity (GM4)

## What will I learn about in this unit?

Pythagoras' Theorem

Trigonometry in right-angled triangles

Exact values in surd form

### Where does this lead?

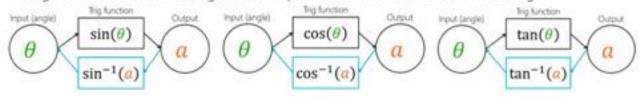
Advanced length and area (GM9)

Trigonometric graphs (A13)

A Level mathematics, physics, engineering and more

## Key words and symbols: what I need to say and write accurately

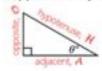
The trigonometric functions have angles as their input. Their inverse functions are written using 1.



The hypotenuse is the longest side in a right-angled triangle, opposite the right angle.

The other two sides are called the adjacent and opposite. They change depending on the angle you look at.





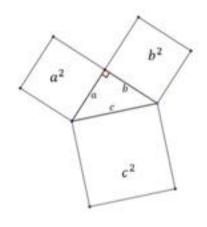


# Fingertip facts: what I need to learn by heart

## Pythagoras' Theorem

For any right-angled triangle,

$$a^2 + b^2 = c^2$$



## Trigonometry

Three equations:

$$sin(\theta) = \frac{opposite}{hypotenuse}$$
  $cos(\theta) = \frac{adjacent}{hypotenuse}$   $tan(\theta) = \frac{opposite}{adjacent}$ 

A mnemonic to help you remember the equations:

The exact values of sine, cosine and tangent for key angles:

Angle, θ	30°	45°	60°
$\sin \theta$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos \theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$\tan \theta$	$\frac{1}{\sqrt{3}}$	1	$\frac{\sqrt{3}}{1} = \sqrt{3}$

# What do I need to remember from before?

Angle facts (GM2)

Area and perimeter (GM3)

Rearranging formulae (A5)

## What will I learn about in this unit?

Naming the parts of a circle

Finding the area and perimeter of circles and sectors of circles

Finding angles in circles using circle theorems

## Where does this lead?

Surface area and volume (GM8)

Advanced length and area (GM9)

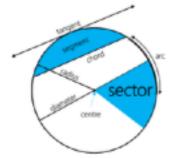
## Key words and symbols: what I need to say and write accurately

The <u>circumference</u> is the perimeter of a circle

We use C for 'circumference'.



We use d for 'diameter' and r for 'radius'.



Semicircle A half circle Quadrant A quarter circle

Concentric circles share the same centre

# Fingertip facts: what I need to learn by heart

 $\pi \approx 3.14$ 

Circle formulae

Circumference:  $C = \pi d$  or  $C = 2\pi r$ 

Area:

 $A=\pi r^2$ 

Sector formulae

We use l for 'arc length'.

Arc length:  $l = \frac{\theta}{360}$ 

Sector area:  $A = \frac{\theta}{360} \pi r^2$ 

### Circle theorems



A radius and a tangent are perpendicular.



The angle at the centre is twice the angle at the circumference (subtended by the same arc).



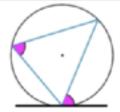
The angle in a semicircle is 90°.



Angles at the circumference subtended by the same arc are equal. (Angles in the same segment are equal).



Opposite angles in a cyclic quadrilateral are sum to 180°.



The Alternate Segment Theorem.

## What do I need to remember from before?

Percentage multipliers (NP8, NP10)

Proportional reasoning (NP10)

Cartesian graphs (A6, A9, A10)

Manipulating algebra (A5, A10)

# What will I learn about in this unit?

Reverse percentages

Simple interest

Direct and inverse proportion

Compound units

Combining ratios

### Where does this lead?

Compound interest (NP16)

Proportion with squares, cubes and roots (NP16)

Instantaneous and average rates of change (A16)

Areas under graphs (A16)

## Key words and symbols: what I need to say and write accurately

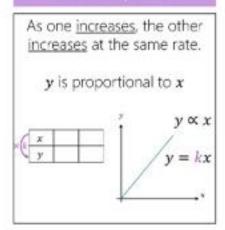
Word	Explanation		
p.a.	per annum, Latin for "every year"		
direct proportion	as one amount increases, the other increases at the same rate		
k	the constant of proportionality, which is the rate of change		
oc	is proportional to		
inverse proportion	as one amount increases, the other decreases at the same rate		
density	a measure of how spread out particles in an object are		
pressure	a measure of how spread out a force is		
speed	a measure of distance per unit of time		
compound units	a measure combining two other measures		

## Fingertip facts: what I need to learn by heart

To calculate a percentage: original × multiplier = new

To return to the original whole: original =  $\frac{\text{new}}{\text{multiplier}}$ 

# Direct Proportion



# Inverse Proportion

As one increases, the other decreases at the same rate.

y is proportional to the reciprocal of x  $y \propto \frac{1}{x}$   $y = \frac{k}{x}$ 

$$Density = \frac{mass}{volume}$$

$$Speed = \frac{distance}{time}$$

## What do I need to remember from before?

Angle facts on lines, parallel lines and in polygons (GM2)

Bearings (GM2)

Constructions and loci (GM1)

Symmetry (GM2)

### What will I learn about in this unit?

Interior and exterior angles in polygons

Reasoning with angle problems

Types and properties of polyhedra

Representing 3D shapes: drawing, isometric drawing, plans and elevations, nets

Constructions and loci problems

#### Where does this lead?

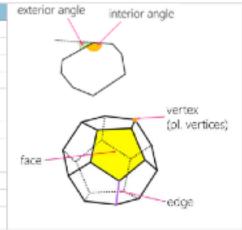
Surface area and volume (GM8)

Advanced length and area (GM9)

Solving geometric problems (GM11)

## Key words and symbols: what I need to say and write accurately

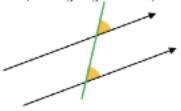
Word	Explanation
polyhedron	a 3D solid with flat, polygon faces (no curved faces)
prism	a polyhedron with a constant cross-section
pyramid	a polyhedron with a decreasing cross-section to the apex
plan	the view when you look down at a 3D shape from above
elevation	the view when you look at a 3D shape from the front or the side
net	what you see when you unfold a 3D shape
bisector	'bisect' means 'cut in half'. A bisector is a line that cuts another in half.
perpendicular	perpendicular lines meet at a right angle.
equidistant	equidistant means an equal distance from two points or lines.
locus (pl. loci)	The path of all points that fit a condition.



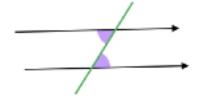
# Fingertip facts: what I need to learn by heart

A bearing is measured clockwise from North and reported with three figures, e.g. 070°.

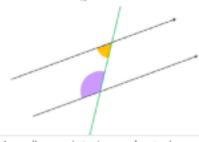
Corresponding angles are equal



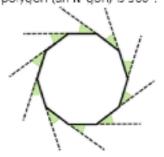
Alternate angles are equal



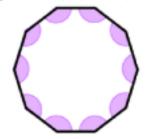
Co-interior angles sum to 180°.



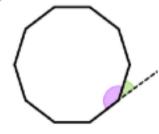
The sum of the exterior angles in any polygon (an n-gon) is 360°.



The sum of the interior angles in any polygon (an n-gon) is  $180(n-2)^{\circ}$ .



An adjacent interior and exterior angle sum to 180°.



GM7 2

# What do I need to remember from before?

Median (NP1, SP1) Mean (NP2, SP1) Mode (SP1)

Range (NP2, SP1) Types of data (SP1)

Comparing summary statistics (SP1)

### What will I learn about in this unit?

Presenting and analysing sets of continuous data.

Estimate measures of location and spread for continuous data

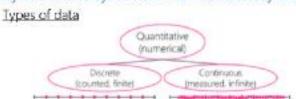
Comparing continuous data sets through charts and summary statistics.

### Where does this lead?

Advanced data analysis from histograms and frequency polygons (SP6)

Advanced statistical analysis (GCSE Statistics; A Level Statistics: Core Maths)

## Key words and symbols: what I need to say and write accurately



Symbols

 $\overline{x}$  – 'the mean'

Σ – 'the sum of (this is the capital Greek letter 'sigma')

Measures of location (capturing where the data set is)

Lower
Quartile (Q<sub>1</sub>)
one-quarter of
the way through
the data set

Me
(C)
mice

Median
(Q2)
middle data
point

Upper Quartile (Q<sub>3</sub>) three-quarters of the way through the data set

Mean total shared equally

Mode most frequent data point Measures of spread (capturing how wide the data set is)

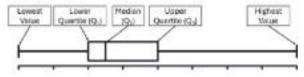
Range spread of all data points

Interquartile Range spread of the points between  $Q_1$  and  $Q_2$ 

A <u>class interval</u> is a group of data, e.g.  $2 \le x < 5$ .

# Fingertip facts: what I need to learn by heart

#### How to draw a boxolot



#### How to find the position of the median and quartiles.

For discrete data:

For continuous data:

 $Q_1$  is in position  $\frac{n+1}{4}$ 

 $Q_1$  is in position  $\frac{n}{4}$ 

 $Q_Z$  is in position  $\frac{n+1}{2}$ 

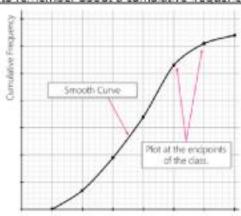
 $Q_2$  is in position  $\frac{h}{a}$ 

 $Q_3$  is in position  $\frac{3(n+1)}{4}$ 

 $Q_3$  is in position  $\frac{3\pi}{4}$ 

(where n is the size of the data set)

# What to remember about a cumulative frequency curve



### How to find the mean from a frequency table

 $\bar{x} = \frac{\sum f^x}{\sum f'}$  where x is a value in a data set (or the midpoint of a class) and f its frequency.

## What do I need to remember from before?

Probability, Venn diagrams and working systematically (SP5)

Linear inequalities (A8)

Types of number (NP4)

# What will I learn about in this unit?

Thinking logically

Sets, subsets and set notation

Sets on Venn diagrams

Probability of A AND/OR B

#### Where does this lead?

Advanced probability problems with independent or dependent events (SP7)

Algebraic proof (A17)

Statistics in Core Maths and A-Level Maths

## Key words and symbols: what I need to say and write accurately

ξ

The universal set. All elements being considered.



 $A = {...}$ 

Set A.



A'

The complement of A. Elements not in A.



 $A \cap B$ 

The intersection of A and B.

Elements in both  $A \underline{AND} B$ .



 $A \cup B$ 

The <u>union</u> of A and B.

Elements in either  $A \bigcirc R$ .



 $A \subset B$ 

A is a subset of B.

Every element in A is also in B.



The solution set to an inequality can be written in set notation. For instance,  $\{x : x > 3\}$  reads "the set of all x such that x is greater than 3."

# Fingertip facts: what I need to learn by heart

If events A and B are <u>mutually exclusive</u>,  $P(A \cup B) = P(A) + P(B)$ .

If events A and B are independent,  $P(A \cap B) = P(A) \times P(B)$ .

If events A and B are not mutually exclusive  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ .

SP5 2

## What do I need to remember from before?

Product of prime factors (NP4)

HCF and LCM (NP3)

Equivalent fractions and decimals (NP8)

Bounds and error (NP9)

# What will I learn about in this unit?

Uses of the product of prime factors

Types of rational number

Error in measurement and calculations

### Where does this lead?

Number theory (after A-Level)

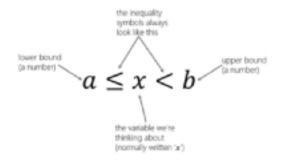
Rationalising fractions (NP15)

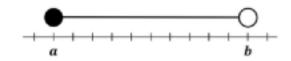
Understanding accuracy in advanced maths, engineering and science (A-Level and beyond)

## Key words and symbols: what I need to say and write accurately

Word/phrase	Explanation
highest common factor (HCF)	the biggest number that is a factor of all the numbers in question e.g. $HCF(16, 20) = 4$
lowest common multiple (LCM)	the smallest number that is a multiple of all the numbers in question e.g. $LCM(16, 20) = 80$
rational number	any number that can be written as a fraction, including integers, terminating and recurring decimals
terminating decimal	a number whose decimal places stop, e.g. 0.2 or 1.81423
recurring decimal	a number whose decimal places continue forever in a repeating pattern, e.g. $0.\hat{2}=0.222222$ or $0.5\hat{6}0\hat{8}=0.5608608608608$
truncate	always round down

An <u>error interval</u> uses inequalities to show the range of values a number could be within. We can show it with inequalities and on a number line.





## Fingertip facts: what I need to learn by heart

The fraction and decimal equivalents for ninths and thirds.

Fraction	1 9	2 9	$\frac{3}{9} = \frac{1}{3}$	4 9	<u>5</u>	$\frac{6}{9} = \frac{2}{3}$	7 9	8 9	$\frac{9}{9} = 1$
Decimal	0. i	0.2	0. 3	0.4	0.5	0.6	0. 7	0.8	$0.\dot{9}=1$

The prime numbers less than 100:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

NP14 2

## What do I need to remember from before?

Adding and subtracting expressions (A2)

Multiplying and dividing expressions (A3)

Factorising expressions (A3)

Simplifying fractions (NP7)

Rearranging formulae (A5)

# What will I learn about in this unit?

Rules of indices: multiplication, division and exponentiation

Multiplying multiple brackets

Factorising guadratics

Simplifying algebraic fractions

Rearranging more complex formulae

### Where does this lead?

Quadratic graphs and equations (A12, A14)

Working with all types of nonlinear functions (A15)

> Operating with algebraic fractions (A17)

Algebraic proof by deduction (A17)

A-Level mathematics

# Key words and symbols: what I need to say and write accurately

Word	Explanation
polynomial	an expression containing only numbers and non-negative powers of $x$ . e.g. $12x + 7$ , $12 + 56x - x^2$ , $12 + 56x - x^2 + 11x^3$ .
binomial	an expression with two terms. e.g. $x = 1$ , $5x + 6$ , $12x^2 + 7x$ .
=	"is identical to". Used to show an identity – when two expressions are identical for every value of $x$ , perhaps just written in a different way. e.g. $x^2 + 5x + 6 \equiv (x + 3)(x + 2)$ or $5x - 3x \equiv 2x$

Types of polynomial	General expanded form	Example (expanded and factorised)
Constant, x0, no brackets	а	5
Linear, $x^1$ , up to one bracket	ax + b	$4x + 10 \equiv 2(2x + 5)$
Quadratic, $x^2$ , up to two brackets	$ax^2 + bx + c$	$2x^2 + 7x + 3 \equiv (2x + 1)(x + 3)$
Cubic, $x^3$ , up to three brackets	$ax^3 + bx^2 + cx + d$	$x^3 + 3x^2 - 6x - 8 \equiv (x+1)(x-2)(x+4)$

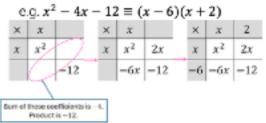
# Fingertip facts: what I need to learn by heart

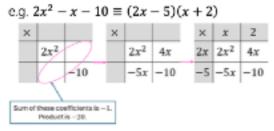
### The index laws

- 1. When we <u>multiply</u> powers with the <u>same base</u>, we can <u>add their exponents</u>.  $x^7 \cdot x^3 = x^{10}$
- 2. When we <u>divide</u> powers with the <u>same base</u>, we can <u>subtract their exponents</u>.  $\frac{x^2}{x^3} = x^4$
- 3. When we find a power of a power, we can multiply the exponents together.  $(x^2)^3 = x^6$

# Factorising a quadratic, $ax^2 + bx + c$

Look for two numbers whose sum is b and whose product is ac.





Difference of two squares e.g.  $x^2 - 9 \equiv (x - 3)(x + 3)$  or  $16x^2 - 49 \equiv (4x - 7)(4x + 7)$ 

## What do I need to remember from before?

Substitution (A5)

Linear and quadratic graphs (A6)

Factorising quadratics (A11)

# What will I learn about in this unit?

Solving quadratic equations using graphs, factorising, the quadratic formula and completing the square

Sketching graphs of quadratics using their key features

#### Where does this lead?

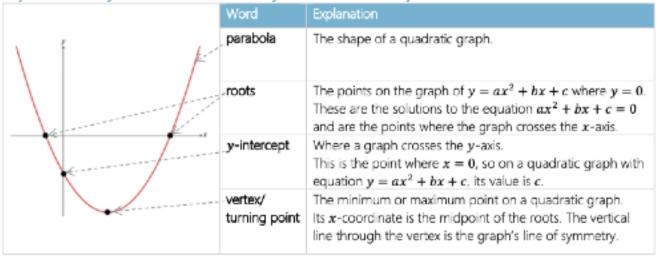
Quadratic sequences (A13)

Advanced quadratic equations (A14)

Quadratic inequalities (A14)

A-Level Mathematics

## Key words and symbols: what I need to say and write accurately



# Fingertip facts: what I need to learn by heart

### The quadratic formula

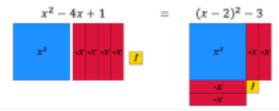
Every quadratic equation of the form  $ax^2 + bx + c = 0$  can be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Completing the square

Every quadratic expression of the form  $x^2 + bx + c$  can be written as an adjustment to a perfect square,  $(x + p)^2 + q$ .

e.g.  $x^2 - 4x + 1$  is 3 less than the perfect square  $(x - 2)^2$ .



# Deciding how to solve a quadratic

If the quadratic can be factorised, do so to solve.

If the quadratic *cannot* be factorised, you should either complete the square (which is easier when a = 1) or use the quadratic formula.

A12

## What do I need to remember from before?

Measuring length, area and volume (NP1)

Area of shapes (GM3)

Types of solids and their nets (GM7)

# What will I learn about in this unit?

Surface area and volume of cubes and cuboids, prisms, cylinders, spheres, pyramids, cones and frustums

Converting between square and cubic units

Surface area and volume of similar solids

### Where does this lead?

Pythagoras and trigonometry in 3D (GM9)

Solving geometric problems, including density and pressure (GM11)

A Level Mathematics

Construction, engineering and architecture

## Fingertip facts: what I need to learn by heart

The concepts of surface area and volume are the same for all solids, but some have specific formulae to learn.

Solid	Surface area (sum of the area of all the faces)	Volume (space inside the solid)	Unit conversions
Cuboid		length × width × height = lwh	When converting between square units, we square the length conversion factor.  e.g. 1 m <sup>2</sup> = 100 <sup>2</sup> cm <sup>2</sup> = 10 000 cm <sup>2</sup>
Prism		area of cross-section × length	When converting between <u>cubic</u> <u>units</u> , we <u>cube</u> the length conversion factor.
Cylinder	area of two circles + curved surface area = $2\pi r^2 + 2\pi rh$ = $2\pi r^2 + \pi dh$	area of cross-section	e.g. 1 m <sup>3</sup> = 100 <sup>3</sup> cm <sup>3</sup> = 1 000 000 cm <sup>3</sup>
Sphere	curved surface area $= 4\pi r^2$	$\frac{4}{3}\pi r^3$	Similar solids The relationship between scale factors of length (1D), area (2D)
Pyramid		$\frac{1}{3}$ volume of a prism with same base and height	and volume (3D):
Cone	base + curved surface area $= \pi r^2 + \pi r l$	$rac{1}{3}$ volume of a cylinder with same base and height $=rac{1}{3}\pi r^2 h$	Volume scale factor, k³ Area scale factor, k²
Frustum	curved surface area (large cone – small cone) + two circles = $\pi RL - \pi rl + \pi R^2 + \pi r^2$	volume of large cone – volume of small cone $= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$	

GM8 2

# What do I need to remember from before?

Substitution (A5)

Linear sequences and their  $n^{th}$  term (A7)

Non-linear sequences (A7)

Quadratic expressions (A11)

# What will I learn about in this unit?

The n<sup>th</sup> term of quadratic and geometric sequences

Recurrence relations

Solving problems with sequences

#### Where does this lead?

A-Level Maths (advanced work with arithmetic and geometric progressions)

Exponential growth and decay (NP16)

## Key words and symbols: what I need to say and write accurately

Word/Symbol	Explanation
$u_n$	The $n^{\text{th}}$ term of a sequence
linear sequence	A sequence where the difference between terms is constant (doesn't change). e.g. 5, 7, 9, 11, (the difference is 2) or 10, 7, 4, 1, (the difference is $-3$ ).
quadratic sequence	A sequence where the differences between terms form a linear sequence. e.g. 1, 4, 9, 16, 25, (the differences are 3, 5, 7, 9,, which is itself a linear sequence).
geometric sequence	A sequence where there is a constant multiplier between terms. e.g. 1, 2, 4, 8, 16, (each term is multiplied by 2 to get the next)
Fibonacci-style sequence	A sequence where each term is the sum of the previous two. e.g. 1, 4, 5, 9, 14, 23,
recurrence relation	A formula that defines a term or terms of a sequence in relation to a previous term or terms. e.g. $u_{n+1} = 2u_n$ ; $u_1 = 5$ mean that each term is double the previous and you start at 5

## Fingertip facts: what I need to learn by heart

The sequence of square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

The sequence of cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, ...

The triangular (or triangle) numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, ...

The Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

The nth term of a quadratic sequence is made up of  $an^2$  (where a is half the second difference) and bn + c which is a linear sequence. To find this nth term, subtract  $an^2$  from the terms in the quadratic sequence to get a linear sequence, then find the nth term of what you get.

e.g. For the sequence 6, 19, 38, 63, 94, ... we get  $3n^2 + 4n - 1$  like this:

Position, n	1	2	3	4	5	n
Sequence, $u_n$	6	19	38	63	94	$3n^2 + 4n - 1$
$an^2 (a=3)$	(3)	(12)	(27)	48	75	$3\eta^2$
Difference	(3)	(7)	(11)	15	19	4n-1

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