My mathematical journey

What do I need to remember from before?

Multiplying to scale (NP3, NP10)

Reflective and rotational symmetry (GM2)

Properties of shapes (GM2)

What will I learn about in this unit?

Congruence and similarity

Congruent transformations: translation, reflection, rotation

Similar transformations: enlargement

Where does this lead?

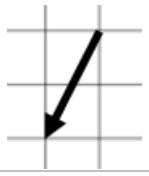
Trigonometry (GM5)

Area and volume in similar solids (GM8)

Problems with congruence and similarity (GM11)

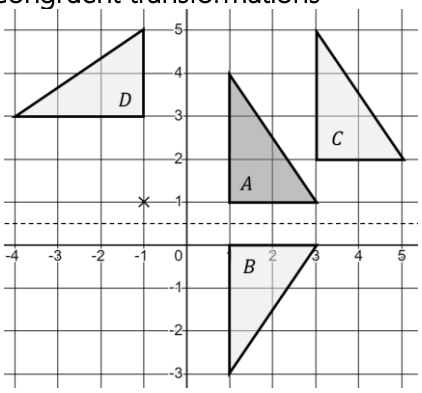
Transforming graphs (A15)

Key words and symbols: what I need to say and write accurately

Word	Explanation
congruent, ≅	identical in size and shape, but not necessarily orientation or direction
transformation	a mathematical change, using translation, rotation, reflection or enlargement
image	a shape *after* a transformation has happened
vector	a mathematical object that tells you how far to move and in what direction it can be shown with an arrow or with column notation e.g. this arrow and column vector both communicate "one left, two down" $\begin{pmatrix}-1\\-2\end{pmatrix}$ 
similar	same shape, all angles the same, but one an enlargement of the other (all corresponding sides in the same ratio)

Fingertip facts: what I need to learn by heart

Congruent transformations



A to B is a reflection in the line $y = \frac{1}{2}$.

A to C is a translation by the vector $\begin{pmatrix}2\\1\end{pmatrix}$

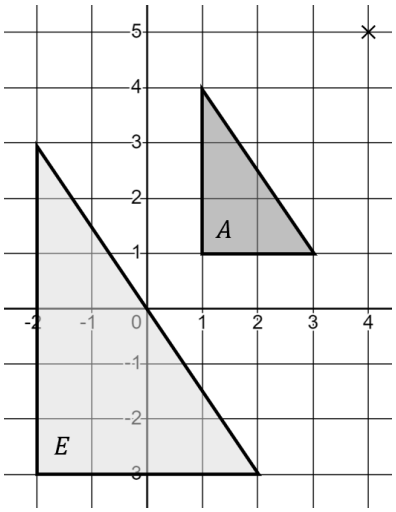
A to D is a rotation 90° anticlockwise around the centre $(-1, 1)$.

Reflections need a mirror line.

Translations need a vector.

Rotations need an angle, direction and centre.

Similar transformations



A to E is an enlargement of scale factor 2 from $(4, 5)$.

Enlargements need a scale factor and centre.

My mathematical journey

What do I need to remember from before?

Plotting graphs (A6)

$y = mx + c$ (A6)

Finding gradient (A6)

Ratio tables and direct proportion (NP10, NP11)

What will I learn about in this unit?

Reading, drawing and interpreting graphs used in various contexts

Finding speed

Distance-time and speed-time graphs

Where does this lead?

Compound units: speed, density, pressure and more (NP13)

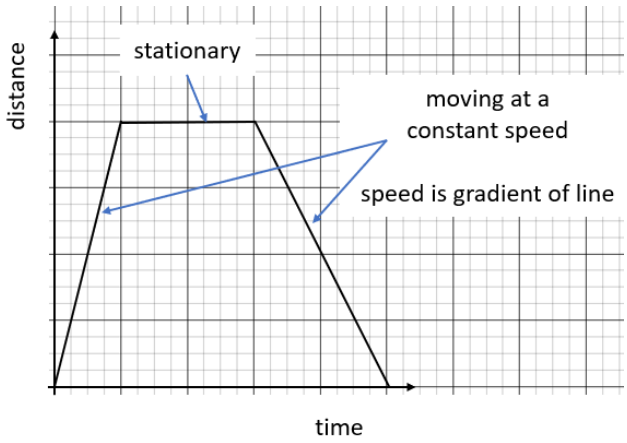
Gradient of and area under of non-linear graphs (A16)

Key words and symbols: what I need to say and write accurately

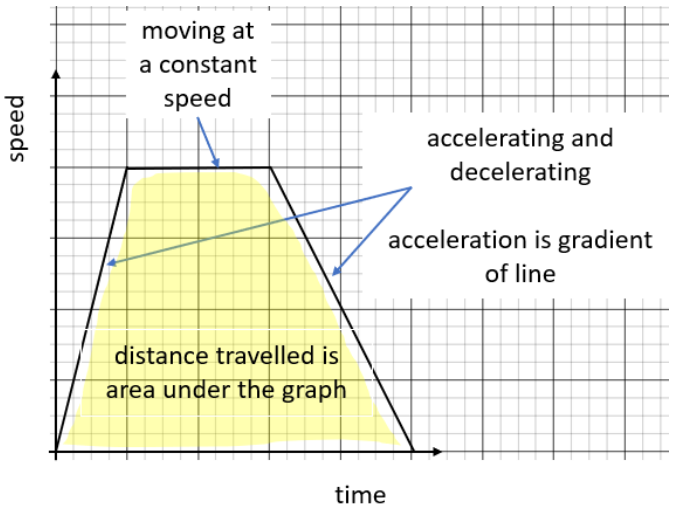
Word	Explanation
gradient	the steepness of a line: for every one unit right, it is the number of units up/down. On a contextual graph, the gradient represents the change in the vertical quantity *per one unit* of the horizontal quantity e.g. the exchange rate between two currencies, or the cost per unit time.
y-intercept	where a graph crosses the y-axis On a contextual graph, the y-intercept represents the value of the vertical quantity when the horizontal quantity is 0 e.g. a fixed or standing charge
speed	a quantity that combines distance and time, working out the distance travelled *per one unit* of time. e.g. miles in 1 hour (miles per hour, mph), or metres in 1 second (metres per second, m/s)

Fingertip facts: what I need to learn by heart

A distance-time graph



A speed-time graph



My mathematical journey

What do I need to remember from before?

Solving linear equations (A4)

Rearranging formulae (A5)

$y = mx + c$ (A6)

Linear inequalities (A8)

What will I learn about in this unit?

Solving problems with graphs

Finding lines parallel and perpendicular to others

Solving simultaneous equations

Plotting inequalities in two dimensions

Where does this lead?

Quadratic graphs (A12)

Non-linear simultaneous equations, non-linear inequalities (A14)

Tangents and normal (A15)

Gradient of a curve (A16)

Using graphs to represent complex problems (A Level Maths)

Key words and symbols: what I need to say and write accurately

Word	Explanation
y-intercept	where a graph crosses the y-axis
x-intercept or root	where a graph crosses the x-axis
satisfy	a number satisfies an equation when it solves the equation
gradient	the steepness of a line
parallel	describing two lines that have the same gradient, so will never intersect
perpendicular	describing two lines that meet at right angles to each other
to intersect	to cross – we say two lines intersect
simultaneously	at the same time
∥	*is parallel to*
⊥	*is perpendicular to*
region	an area on a graph
boundary	a line that marks the edge of a region

Fingertip facts: what I need to learn by heart

If a point (x, y) lies on a line, its coordinates can be substituted for y and x in the equation of the line and they will satisfy the equation.

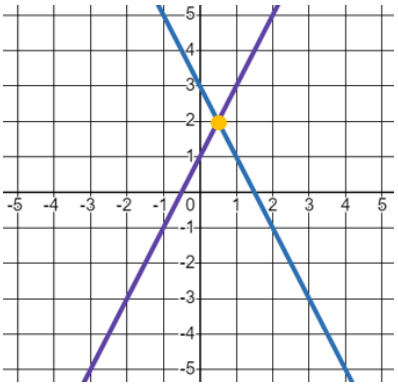
The gradient of a line is the amount up/down it moves for every one unit right. We can work this out by calculating the ratio $\frac{\text{vertical}}{\text{horizontal}}$ between two points on the line.

The gradients of parallel lines are the same.

The gradients of perpendicular lines are the negative reciprocal of each other: $+\frac{a}{b} \perp -\frac{b}{a}$

On the y-axis, $x = 0$. On the x-axis, $y = 0$.

When we solve simultaneous equations, we find the point of intersection of graphs of the two equations.

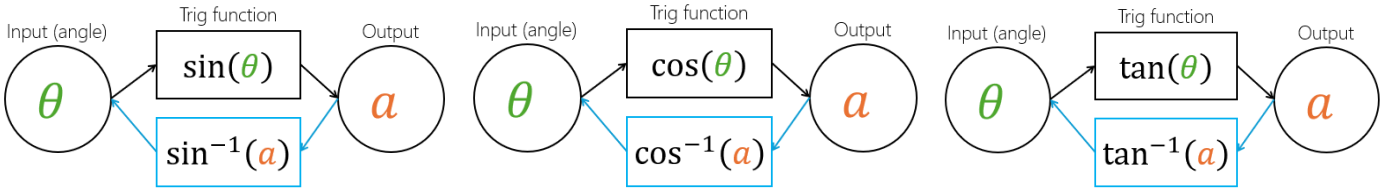


My mathematical journey

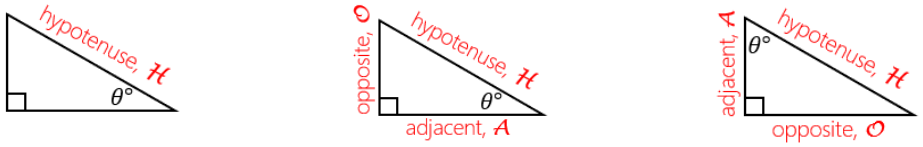
What do I need to remember from before?	What will I learn about in this unit?	Where does this lead?
Powers and roots (NP4)	Pythagoras' Theorem	Advanced length and area (GM9)
Substitution and rearranging a formulae (A5)	Trigonometry in right-angled triangles	Trigonometric graphs (A13)
Angles in triangles (GM2)	Exact values in surd form	A Level mathematics, physics, engineering and more
Similarity (GM4)		

Key words and symbols: what I need to say and write accurately

The trigonometric functions have angles as their input. Their inverse functions are written using $^{-1}$.



The <u>hypotenuse</u> is the longest side in a right-angled triangle, opposite the right angle.
The other two sides are called the <u>adjacent</u> and opposite. They change depending on the angle you look at.

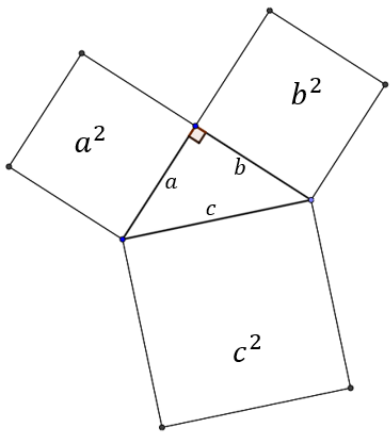


Fingertip facts: what I need to learn by heart

<u>Pythagoras' Theorem</u>

For any right-angled triangle,

$$a^2 + b^2 = c^2.$$



<u>Trigonometry</u>

Three equations:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

A mnemonic to help you remember the equations:

$$\text{S}^{\text{O}}\text{H} \quad \text{C}^{\text{A}}\text{H} \quad \text{T}^{\text{O}}\text{A}$$

The exact values of sine, cosine and tangent for key angles:

Angle, θ	30°	45°	60°
$\sin\theta$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\frac{\sqrt{3}}{1} = \sqrt{3}$

My mathematical journey

What do I need to remember from before?

Number lines (NP1, 2, 3, and 6)

Decimals (NP1, 2, and 3)

Fractions (NP7)

Frequency tables (SP1)

What will I learn about in this unit?

Systematic listing and the product rule

Experimental and theoretical probability

Probability diagrams

Where does this lead?

Sets and Venn diagrams (SP5)

Sampling and data analysis (SP6)

Advanced probability problems (SP7)

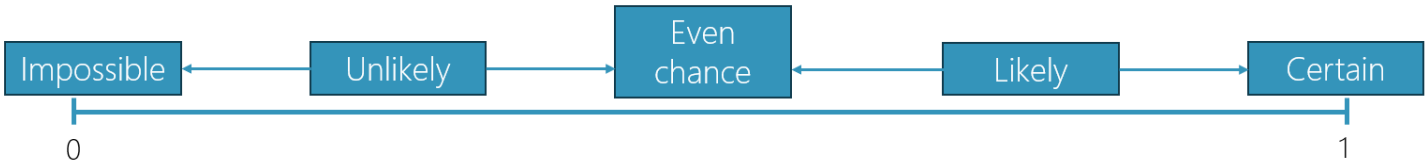
Key words and symbols: what I need to say and write accurately

Word	Explanation
systematic	working in an organised way
relative frequency	the proportion of times something happens
outcome	a result we could get from a probability experiment e.g. rolling a fair six-sided die gives the outcomes 1, 2, 3, 4, 5, and 6
event	one or more outcomes e.g. rolling a square number on a die
fair	all outcomes are equally likely
biased	some outcomes are more likely than others
mutually exclusive	events which cannot happen at the same time
independent	If events are independent, they do not influence or affect each other. e.g. if I flip a coin twice, the outcome of the first flip has no effect on the outcome of the second flip: the events are independent.

Fingertip facts: what I need to learn by heart

$$P(\text{event}) = \frac{\text{outcomes we want}}{\text{total outcomes}}$$

Probabilities can be represented by a number between 0 and 1.



The sum of all mutually exclusive events is 1.

If events A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B)$

$P(\text{not } A) = 1 - P(A)$. This can also be written $P(A') = 1 - P(A)$

My mathematical journey

What do I need to remember from before?

Angle facts (GM2)

Area and perimeter (GM3)

Rearranging formulae (A5)

What will I learn about in this unit?

Naming the parts of a circle

Finding the area and perimeter of circles and sectors of circles

Finding angles in circles using circle theorems

Where does this lead?

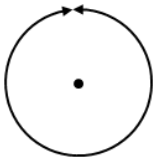
Surface area and volume (GM8)

Advanced length and area (GM9)

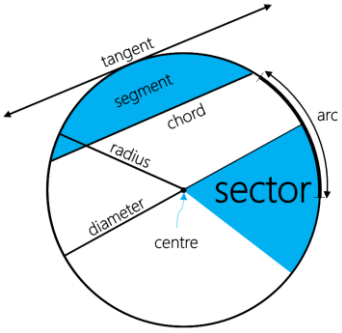
Key words and symbols: what I need to say and write accurately

The <u>circumference</u> is the perimeter of a circle.

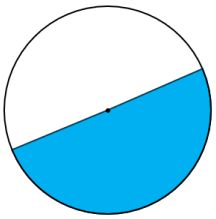
We use C for 'circumference'.



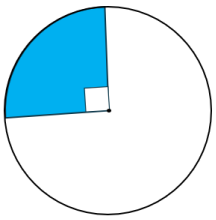
We use d for 'diameter' and r for 'radius'.



<u>Semicircle</u>
A half circle



<u>Quadrant</u>
A quarter circle

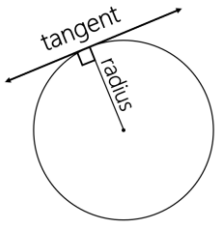
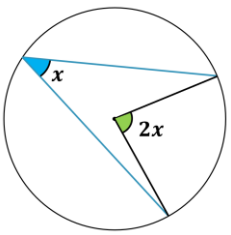
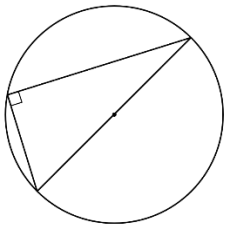
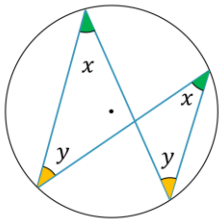
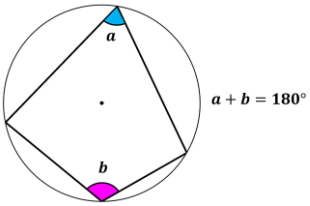
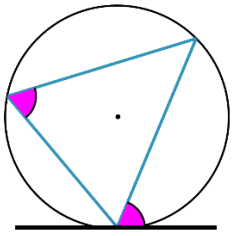


<u>Concentric</u> circles share the same centre

Fingertip facts: what I need to learn by heart

$\pi \approx 3.14$	<u>Circle formulae</u> Circumference: $C = \pi d$ or $C = 2\pi r$ Area: $A = \pi r^2$	<u>Sector formulae</u> We use l for 'arc length'. Arc length: $l = \frac{\theta}{360}\pi d$ Sector area: $A = \frac{\theta}{360}\pi r^2$

<u>Circle theorems</u>

	A radius and a tangent are perpendicular.		The angle at the centre is twice the angle at the circumference (subtended by the same arc).
	The angle in a semicircle is 90°.		Angles at the circumference subtended by the same arc are equal. (Angles in the same segment are equal).
	Opposite angles in a cyclic quadrilateral are sum to 180°.		The Alternate Segment Theorem.

My mathematical journey

What do I need to remember from before?

Angle facts on lines, parallel lines and in polygons (GM2)

Bearings (GM2)

Constructions and loci (GM1)

Symmetry (GM2)

What will I learn about in this unit?

Interior and exterior angles in polygons

Reasoning with angle problems

Types and properties of polyhedra

Representing 3D shapes: drawing, isometric drawing, plans and elevations, nets

Constructions and loci problems

Where does this lead?

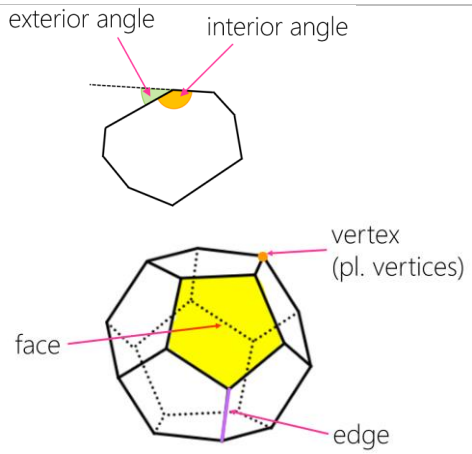
Surface area and volume (GM8)

Advanced length and area (GM9)

Solving geometric problems (GM11)

Key words and symbols: what I need to say and write accurately

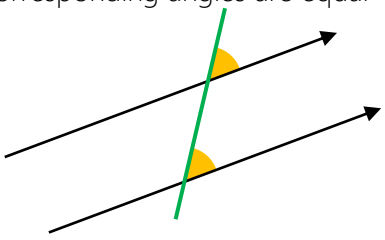
Word	Explanation
polyhedron	a 3D solid with flat, polygon faces (no curved faces)
prism	a polyhedron with a constant cross-section
pyramid	a polyhedron with a decreasing cross-section to the apex
plan	the view when you look down at a 3D shape from above
elevation	the view when you look at a 3D shape from the front or the side
net	what you see when you unfold a 3D shape
bisector	'bisect' means 'cut in half'. A bisector is a line that cuts another in half.
perpendicular	perpendicular lines meet at a right angle.
equidistant	equidistant means an equal distance from two points or lines.
locus (pl. loci)	The path of all points that fit a condition.



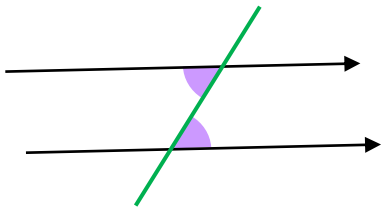
Fingertip facts: what I need to learn by heart

A bearing is measured clockwise from North and reported with three figures, e.g. 070°.

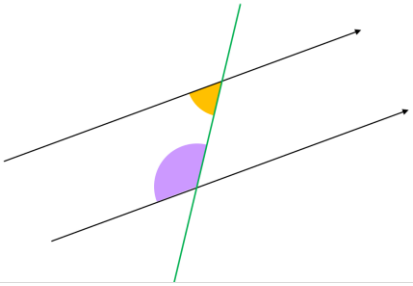
Corresponding angles are equal



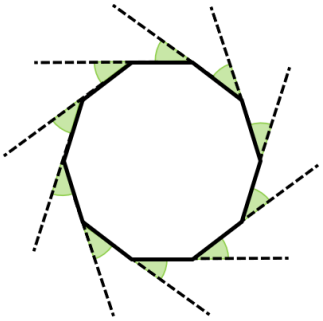
Alternate angles are equal



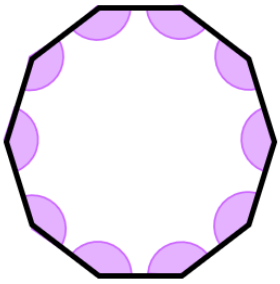
Co-interior angles sum to 180°



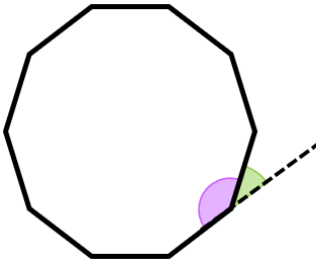
The sum of the exterior angles in any polygon (an n-gon) is 360°.



The sum of the interior angles in any polygon (an n-gon) is $180(n-2)°$.



An adjacent interior and exterior angle sum to 180°.



My mathematical journey

What do I need to remember from before?

Adding and subtracting expressions (A2)

Multiplying and dividing expressions (A3)

Factorising expressions (A3)

Simplifying fractions (NP7)

Rearranging formulae (A5)

What will I learn about in this unit?

Rules of indices: multiplication, division and exponentiation

Multiplying multiple brackets

Factorising quadratics

Simplifying algebraic fractions

Rearranging more complex formulae

Where does this lead?

Quadratic graphs and equations (A12, A14)

Working with all types of non-linear functions (A15)

Operating with algebraic fractions (A17)

Algebraic proof by deduction (A17)

A-Level mathematics

Key words and symbols: what I need to say and write accurately

Word	Explanation
polynomial	an expression containing only numbers and non-negative powers of x. e.g. $12x + 7$, $12 + 56x - x^2$, $12 + 56x - x^2 + 11x^3$.
binomial	an expression with two terms. e.g. $x - 1$, $5x + 6$,$12x^2 + 7x$.
$\equiv$	"is identical to". Used to show an identity – when two expressions are identical for every value of x, perhaps just written in a different way. e.g. $x^2 + 5x + 6 \equiv (x + 3)(x + 2)$ or $5x - 3x \equiv 2x$

Types of polynomial	General expanded form	Example (expanded and factorised)
Constant, x^0, no brackets	a	5
Linear, x^1, up to one bracket	$ax + b$	$4x + 10 \equiv 2(2x + 5)$
Quadratic, x^2, up to two brackets	$ax^2 + bx + c$	$2x^2 + 7x + 3 \equiv (2x + 1)(x + 3)$
Cubic, x^3, up to three brackets	$ax^3 + bx^2 + cx + d$	$x^3 + 3x^2 - 6x - 8 \equiv (x + 1)(x - 2)(x + 4)$

Fingertip facts: what I need to learn by heart

The index laws

1. When we multiply powers with the same base, we can add their exponents. $x^7 \cdot x^3 = x^{10}$
2. When we divide powers with the same base, we can subtract their exponents. $\frac{x^7}{x^3} = x^4$
3. When we find a power of a power, we can multiply the exponents together. $(x^2)^3 = x^6$

Factorising a quadratic, $ax^2 + bx + c$

Look for two numbers whose sum is b and whose product is ac.

e.g. $x^2 - 4x - 12 \equiv (x - 6)(x + 2)$

×	x	
x	x^2	
		-12

×	x	
x	x^2	$2x$
	$-6x$	-12

×	x	2
x	x^2	$2x$
-6	$-6x$	-12

Sum of these coefficients is -4.
Product is -12.

e.g. $2x^2 - x - 10 \equiv (2x - 5)(x + 2)$

×		
	$2x^2$	
		-10

×		
	$2x^2$	$4x$
	$-5x$	-10

×	x	2
$2x$	$2x^2$	$4x$
-5	$-5x$	-10

Sum of these coefficients is -1.
Product is -20.

Difference of two squares

e.g. $x^2 - 9 \equiv (x - 3)(x + 3)$ or $16x^2 - 49 \equiv (4x - 7)(4x + 7)$

My mathematical journey

What do I need to remember from before?

Substitution (A5)

Linear and quadratic graphs (A6)

Factorising quadratics (A11)

What will I learn about in this unit?

Solving quadratic equations using graphs, factorising, the quadratic formula and completing the square

Sketching graphs of quadratics using their key features

Where does this lead?

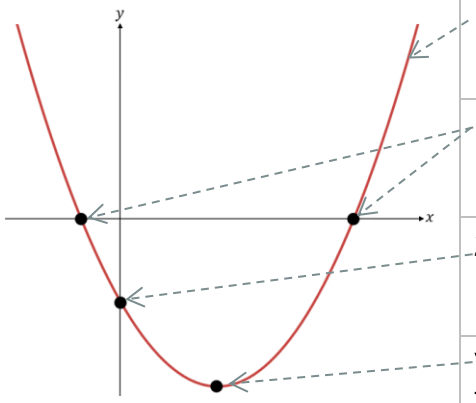
Quadratic sequences (A13)

Advanced quadratic equations (A14)

Quadratic inequalities (A14)

A-Level Mathematics

Key words and symbols: what I need to say and write accurately



Word	Explanation
parabola	The shape of a quadratic graph.
roots	The points on the graph of $y = ax^2 + bx + c$ where $y = 0$. These are the solutions to the equation $ax^2 + bx + c = 0$ and are the points where the graph crosses the x-axis.
y-intercept	Where a graph crosses the y-axis. This is the point where $x = 0$, so on a quadratic graph with equation $y = ax^2 + bx + c$, its value is c.
vertex/ turning point	The minimum or maximum point on a quadratic graph. Its x-coordinate is the midpoint of the roots. The vertical line through the vertex is the graph's line of symmetry.

Fingertip facts: what I need to learn by heart

The quadratic formula

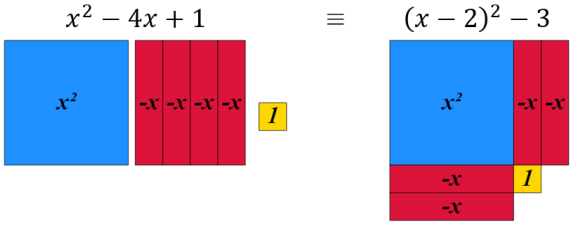
Every quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the square

Every quadratic expression of the form $x^2 + bx + c$ can be written as an adjustment to a perfect square, $(x + p)^2 + q$.

e.g. $x^2 - 4x + 1$ is 3 less than the perfect square $(x - 2)^2$.



Deciding how to solve a quadratic

If the quadratic *can* be factorised, do so to solve.

If the quadratic *cannot* be factorised, you should either complete the square (which is easier when $a = 1$) or use the quadratic formula.

My mathematical journey

What do I need to remember from before?

Substitution (A5)

Linear sequences and their n^{th} term (A7)

Non-linear sequences (A7)

Quadratic expressions (A11)

What will I learn about in this unit?

The n^{th} term of quadratic and geometric sequences

Recurrence relations

Solving problems with sequences

Where does this lead?

A-Level Maths (advanced work with arithmetic and geometric progressions)

Exponential growth and decay (NP16)

Key words and symbols: what I need to say and write accurately

Word/Symbol	Explanation
u_n	The n^{th} term of a sequence
linear sequence	A sequence where the difference between terms is constant (doesn't change). e.g. 5, 7, 9, 11, … (the difference is 2) or 10, 7, 4, 1, … (the difference is -3).
quadratic sequence	A sequence where the differences between terms form a linear sequence. e.g. 1, 4, 9, 16, 25, … (the differences are 3, 5, 7, 9, …, which is itself a linear sequence).
geometric sequence	A sequence where there is a constant multiplier between terms. e.g. 1, 2, 4, 8, 16, … (each term is multiplied by 2 to get the next)
Fibonacci-style sequence	A sequence where each term is the sum of the previous two. e.g. 1, 4, 5, 9, 14, 23, …
recurrence relation	A formula that defines a term or terms of a sequence in relation to a previous term or terms. e.g. $u_{n+1} = 2u_n; u_1 = 5$ mean that each term is double the previous and you start at 5

Fingertip facts: what I need to learn by heart

The sequence of square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, …

The sequence of cube numbers: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, …

The triangular (or triangle) numbers: 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, …

The Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

The nth term of a quadratic sequence is made up of an^2 (where a is half the second difference) and $bn + c$ which is a linear sequence. To find this nth term, subtract an^2 from the terms in the quadratic sequence to get a linear sequence, then find the nth term of what you get.

e.g. For the sequence 6, 19, 38, 63, 94, … we get $3n^2 + 4n - 1$ like this:

Position, n	1	2	3	4	5	n
Sequence, u_n	6	19	38	63	94	$3n^2 + 4n - 1$
an^2 $(a = 3)$	3	12	27	48	75	$3n^2$
Difference	3	7	11	15	19	$4n - 1$